Bureaucratic agency exercise 2

The bureau maximizes utility $X^{\beta}Z^{1-\beta}$ subject to $W(X) \geq B$ and $X = \hat{\alpha}^{\frac{1}{\gamma-1}}$. Using the first constraint, we find that the reported cost is

$$\hat{C}_0 = \frac{1-\gamma}{\gamma}\hat{\alpha}^{\frac{\gamma}{\gamma-1}} \tag{1}$$

and hence the obtained slack becomes

$$Z = \frac{1}{\gamma}\hat{\alpha}^{\frac{\gamma}{\gamma-1}} - \alpha\hat{\alpha}^{\frac{1}{\gamma-1}} - C_0 \tag{2}$$

Hence the bureau chooses $\hat{\alpha}$ to maximize

$$\frac{\beta}{\gamma-1}\ln\hat{\alpha} + (1-\beta)\ln\left(\frac{1}{\gamma}\hat{\alpha}^{\frac{\gamma}{\gamma-1}} - \alpha\hat{\alpha}^{\frac{1}{\gamma-1}} - C_0\right) \tag{3}$$

which yields the FOC

$$\frac{\beta}{\gamma-1}\frac{1}{\hat{\alpha}} + (1-\beta)\frac{\frac{1}{\gamma-1}\hat{\alpha}^{\frac{1}{\gamma-1}} - \frac{1}{\gamma-1}\alpha\hat{\alpha}^{\frac{-\gamma}{\gamma-1}}}{\frac{1}{\gamma}\hat{\alpha}^{\frac{\gamma}{\gamma-1}} - \alpha\hat{\alpha}^{\frac{1}{\gamma-1}} - C_0} = 0 \tag{4}$$

which can be simplified to

$$\left(\frac{\beta}{\gamma} + (1-\beta)\right)\hat{\alpha}^{\frac{\gamma}{\gamma-1}} - \alpha\hat{\alpha}^{\frac{1}{\gamma-1}} = C_0 \tag{5}$$

However, we can't find a closed form solution for $\hat{\alpha}$.