ECON4925 Resource economics, Autumn 2009

Electricity: Uncertainty and constraints on reservoir capacity

We want to compare the case of a known amount of precipitation $\bar{S}$ with an uncertain precipitation with with expected amount equal to $\bar{S}$. We assume there is a constraint K on reservoir capacity in period 2, but that this constraint is not binding in the case of certainty. In the certain case we maximize $U(x_1) + U(\bar{S} - x_1)$, giving

$$U'(x_1^0) - U'(\bar{S} - x_1^0) = 0 \tag{1}$$

where we assume $x_2^0 = \bar{S} - x_1^0 < K$.

With uncertainty we assume S takes the value S^H with probability π and S^L with probability $1-\pi$, satisfying $ES = \pi S^H + (1-\pi)S^L = \bar{S}$. Moreover, we assume that for all relevant values of x_1 we have

$$x_2 = K \quad if \quad S = S^H$$
$$x_2 = S^L - x_1 \quad if \quad S = S^L$$

In the case of uncertainty we therefore maximize $U(x_1) + \left[\pi U(K) + (1-\pi)U(S^L - x_1)\right]$, giving

$$U'(x_1^*) - (1-\pi)U'(S^L - x_1^*) = 0 \tag{2}$$

which implies that

$$U'(x_1^*) - \left[\pi U'(S^H - x_1^*) + (1-\pi)U'(S^L - x_1^*)\right] < 0$$

Assuming that $U''' = 0$, i.e. that we have a linear demand function, the inequality above can be rewritten as

$$U'(x_1^*) - U'([\pi S^H + (1-\pi)S^L] - x_1^*) < 0$$

or

$$U'(x_1^*) - U'(\bar{S} - x_1^*) < 0 \tag{3}$$

Comparing (3) with (1) and remembering that $U'' < 0$ it follows that $x_1^* > x_1^0$. In other words, uncertainty about S in this case implies larger production, and thus a lower price, in the first period than we had with no uncertainty.