Econ4925 – Seminar 2

Fossil fuels & Climate Change

There exists a fossil fuel resource with zero extraction costs and a total initial stock equal to A_0. The representative individual derives utility from extraction of the resource (x_t). In particular, the utility function (in terms of the numeraire good) is:

$$u(x_t) = ax_t - \frac{b}{2}x_t^2 \quad a, b > 0$$

However, extraction and useage of the resource increase the amount of greenhouse gases (S) in the atmosphere. Those decompose only slowly. S develops according to:

$$\dot{S}_t = x_t - \delta S_t \quad \delta \in (0; 1)$$

The harm caused by S is described by the damage function:

$$D(S_t) = kS_t \quad k > 0$$

Suppose that there is no backstop technology, i.e. no substitute. Presume that the time horizon for this problem is very large (N) or infinite (∞).

1. *Show that the socially optimal extraction path is:*

$$x_t = \frac{1}{b}\left[a - \frac{k}{r+\delta} - \lambda_0 e^{rt}\right]$$

2. *How is λ_0 determined?*

3. *Show how this socially optimal outcome can be achieved in a competitive economy with an appropriate tax q on resource extraction.*

CCS

Assume now that it is feasible to use "carbon capture and storage" (CCS) technology to capture the greenhouse gases. Using CCS to an extent h costs c per unit and means that the stock develops according to:

$$\dot{S}_t = x_t - h_t - \delta S_t$$

4. *Under which conditions is it optimal to use CCS? To what extent is it used if it is used at all?*

5. *How does your answer to* (4) *change if the damage function is instead:*

$$D(S_t) = \frac{m}{2} S_t^2 \quad m > 0$$