

## Econ4925 – Seminar 6

### Optimal growth with a natural resource constraint

Consider an economy with an aggregate production function

$$Q_t = K_t^\alpha (M_t R_t)^{1-\alpha}$$

where  $K$  is capital,  $R$  is the use of an exhaustible resource (which can be extracted without costs) and  $M$  is a variable representing the technology level of the economy. All these variables depend on time  $t$ .  $\alpha \in (0, 1)$ .

1. Show that the interest rate in the economy, i.e. the marginal productivity of capital, is constant provided  $\frac{MR}{K}$  is constant.
2. Show that when the interest rate is constant, the resource price, i.e. the marginal productivity of the resource, is proportional to  $M$ .
3. Explain what is meant by intertemporal efficiency.
4. Assume that  $M_t$  grows at a constant rate  $m > 0$ . Show that in this case it is possible to have an intertemporally efficient growth path along which the interest rate is constant and output grows at a constant rate  $g$ .
5. What is the interest rate along the path described in (4)?
6. What is the relationship between the growth rate  $g$  and the saving rate  $s = \frac{\dot{K}}{Q}$  along the path described in (4)?
7. Assume that the population is constant and that social welfare is given by the function

$$\int_0^\infty u(C_t) e^{-\rho t} dt$$

where  $C = Q - \dot{K}$ . Derive the conditions for the optimal outcome, and show under what conditions a growth path of the type described in (4) is optimal (assume that  $\frac{-u''C}{u'}$  is constant). When such a growth path exists, how does the growth rate depend on the parameters in the welfare function?