ECON4925 Resource economics, Autumn 2011

Lecture 10-11: Discounting, growth and sustainability

Reading list:
Perman et al. (2003): Sec. 3.5, Ch. 4
Hoel (1978b: "Consumption growth…"). A more comprehensive but also more difficult treatment is given in Dixit at al.

Outline of lectures:

I. Intertemporal optimization in a one person two period economy: maximize $U(C_1, C_2)$ subject to $C_1 = X - I$ and $C_2 = F(I)$

II. The role of the interest rate in a market economy

III. The special case of $U(C_1, C_2) = u(C_1) + \beta u(C_2)$

IV. Allocations across generations: Who's preferences? Perman 3.5.1

V. Extension to two capital goods: $C_1 = X - I - J$ and $C_2 = F(I, J)$. Intertemporal efficiency implies $F_I(I, J) = F_J(I, J)$. Perman 4.1.1 and 4.1.2

VI. Efficient and optimal growth with a natural resource. Perman 3.5.2. See also below (last two pages).

VII. Definitions of sustainability. Perman 4 and 19.2.3.

VIII. Hartwick's rule: Hoel (1978b): "Consumption growth…" See also below (last page).

IX. Intertemporal efficiency and Hartwick's rule in a general model (see below)

Intertemporal efficiency and Hartwick's Rule in a general model

$v(K,I)$ = utility at a given time, K and I are vectors; $v_K \geq 0$ and $v_I < 0$

Interpretations of $v(K,I)$ are given below.

Max $\int_0^\infty e^{-\rho t} v(K(t),I(t))dt$ s.t. $\dot{K} = I$ gives

(*) $\dfrac{v_{Kj} - \dot{v}_{Ij}}{-v_{Ij}} = \rho$ for all j

Efficiency combined with zero saving (i.e. $\sum_j (-v_{Ij})I_j = 0$) gives v constant.

Proof (in sloppy notation):

$\dot{v} = \Sigma v_K I + \Sigma v_I \dot{I}$

$\Sigma v_I I = 0$ implies $\Sigma v_I \dot{I} = -\Sigma \dot{v}_I I$ so that $\dot{v} = \Sigma(v_K - \dot{v}_I)I$

From efficiency we thus have $\dot{v} = -\rho \Sigma v_I I = 0$

Example 1: a non-renewable resource

$$v(K,S,I,-R) = u(F(K,R) - I)$$

$$v_K = u' F_K \qquad v_I = -u'$$

$$v_S = 0 \qquad v_{-R} = -u' F_R$$

Efficiency condition (*) gives Hotelling rule $\dfrac{\dot{F}_R}{F_R} = F_K$

and also Ramsey rule $F_K = \rho - \dfrac{\dot{u}'}{u'}$

Example 2: Environmental Quality Q as a resource

$$Q = \overline{Q} - S \qquad \dot{S} = E - \delta S \qquad \text{so } E = \delta\overline{Q} - \delta Q - J \quad \text{where } J = \dot{Q}$$

$$v(K,Q,I,J) = u(Q, F(K, \delta\overline{Q} - \delta Q - J) - I)$$

$$v_K = u_C F_K \qquad v_I = -u_C$$

$$v_Q = u_Q - u_C F_E \delta \qquad v_J = -u_C F_E$$

Efficiency condition (*) gives rule for development of optimal emission tax:

$$\dot{F}_E = (F_K + \delta)F_E - \frac{u_Q}{u_C}$$

Efficient and optimal growth with natural resources

Simple optimization problem with constant population and constant technology (ignoring time references most places):

$$\max \int_0^\infty e^{-\rho t} u(c(t))dt$$
$$c = F(K, R) - I$$
$$\dot{K} = I$$
$$\dot{S} = -R \leq 0$$
$$S(t) \geq 0 \quad for\ all\ t$$

Current value Hamiltonian:

$$H = u(F(K, R) - I) + \mu I - \lambda R$$

Sketch of solution:

$$\dot{\mu} = \rho\mu - u' F_K \tag{1}$$
$$\dot{\lambda} = \rho\lambda \tag{2}$$
$$-u' + \mu = 0 \tag{3}$$
$$u' F_R - \lambda = 0 \tag{4}$$

Combining (1) and (3) gives

$$\frac{\dot{\mu}}{\mu} = \rho - F_K$$

Combining (2), (3) and (4) gives

$$\frac{\dot{\mu}}{\mu} = \frac{\dot{\lambda}}{\lambda} - \frac{\dot{F}_R}{F_R} = \rho - \frac{\dot{F}_R}{F_R}$$

Taken together, and using (3), these two equations give

$$\frac{\dot{F}_R}{F_R} = F_K \quad Hotelling\ rule$$
$$F_K = \rho + \alpha(c)\frac{\dot{c}}{c} \quad Ramsey\ rule$$

where $\alpha(c) = \frac{-u''(c)c}{u'(c)} > 0$.

Hartwick's rule

Assume instead of the Ramsey rule that investment in physical capital is always equal to the resource rent:

$$\dot{K} = F_R R$$

Using the Hotelling rule this implies that

$$\dot{I} = \ddot{K} = F_R(\dot{R} + F_K R)$$

Inserting into $c = F(K, R) - I)$ and differentiating gives

$$\dot{c} = F_K \dot{K} + F_R \dot{R} - \dot{I} = 0$$

Hartwick's rule thus gives constant consumption.