ECON 4925 Resource Economics

Lecture note 8, Michael Hoel

Non-renewable resources: Extraction costs and taxes

Extraction costs

Costs depend on time: $c(t)$

If extraction costs depend on time, we as before find

$$p(t) = c(t) + \lambda(t)$$
$$\lambda(t) = \lambda_0 e^{rt}$$

which implies

$$\dot{p} = \dot{c} + \dot{\lambda} = \dot{c} + r\lambda = \dot{c} + r(p - c) \quad (1)$$

If costs are declining sufficiently rapidly, the resource price may therefore decline.

Costs depend on accumulated extraction: $c(A)$:

Consider the dynamic optimization problem (ignoring time references where this cannot cause misunderstanding)

$$\max \int_0^\infty e^{-rt} \left[u(x) - c(A)x \right] dt$$

subject to

$$A(t) = S_0 - S(t)$$

$$\dot{S} = -x$$

$S(0) = S_0$ historically given initial resource stock

$$x(t) \geq 0 \text{ for all } t$$
$$S(t) \geq 0 \text{ for all } t$$

As before, we assume $u(0) = 0$, $u' > 0$, $u'' < 0$ and $u'(0) = b$. We now also assume that $c(A)$ is positive and increasing in A, and that

$c(S_0) > b > c(0)$. The condition $b > c(0)$ means that it is optimal to use some of the resource, while the condition $c(S_0) > b$ means that it is not optimal to use up all of the physically available resource.

The Hamiltonian in this case is

$$H(x, S, \lambda) = u(x) - c(S_0 - S)x - \lambda x$$

It is "obvious" that the condition $c(S_0) > b$ implies that the constraint $S(t) \geq 0$ is not binding for the optimization problem. The optimum conditions are therefore

$$\frac{\partial H}{\partial x} = u'(x) - c(S_0 - S) - \lambda = 0 \text{ for } x > 0 \tag{2}$$

$$\dot{\lambda} = r\lambda - \frac{\partial H}{\partial S} = r\lambda - xc'(S_0 - S) \tag{3}$$

$$Lim_{t \to \infty} e^{-rt} \lambda(t) S(t) = 0 \tag{4}$$

It is useful to see if there exists a stationary solution (S^*, λ^*, x^*) satisfying the optimum conditions. If there is, it is clear from $\dot{S} = -x$ and (3) that $\lambda^* = x^* = 0$. If also S^* is given by $c(S_0 - S^*) = b$ all the optimum conditions are satisfied.

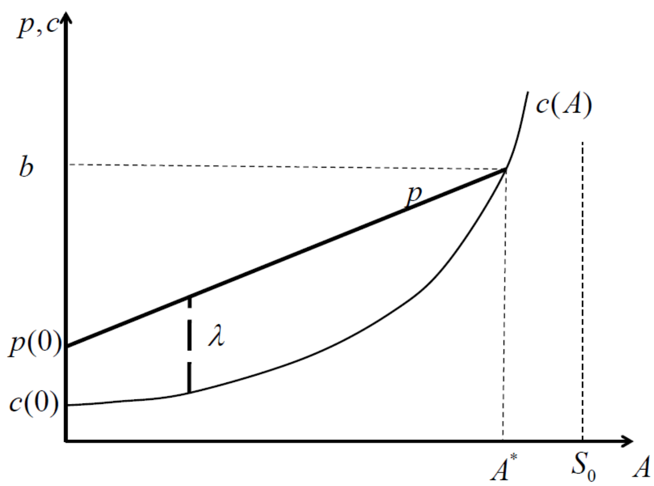
Notice that if $S_0 \neq S^*$ the optimal solution cannot have actually be (S^*, λ^*, x^*) for any time period. The reason for this is that once we are at (S^*, λ^*, x^*) , there is nothing to move the variable away from these values, whether we move backwards or forwards in time. However, the optimal solution will approach (S^*, λ^*, x^*) asymptotically: As long as $c(A) < b$, it is socially beneficial to continue resource extraction, implying that A will grow. This will continue until A gradually reaches its upper limit A^* defined by $c(A^*) = b$, since it is not beneficial to continue extraction for $c(A) > b$, i.e. marginal extraction costs exceeding the marginal utility of the resource.

As long as $x > 0$,we know from (3) that the development of resource rent λ satisfies $\dot{\lambda} < r\lambda$. We do not generally know the sign of $\dot{\lambda}$, although we know that λ must eventually decline towards 0. The price $p = u'(x) =$

$c(A) + \lambda$ must however always rise:

$$\dot{p} = c'\dot{A} + \dot{\lambda} = c'x + r\lambda - xc' = r\lambda = r(p - c(A)) > 0 \tag{5}$$

The figure below illustrates the development of the price path (heavily drawn) as A increases (i.e. as S declines)



Notice that the slope of the p-curve in this diagram is given by

$$\frac{dp}{dA} = \frac{\dot{p}}{\dot{A}} = \frac{r(p - c(A))}{x(p)} \tag{6}$$

and is thus flatter the larger is A (since $c' > 0$) and the lower is p .

Taxes

I consider the following taxes:

1. a constant tax rate τ_π on profit/cash flow
2. a constant tax rate τ_R on gross revenue
3. a constant tax rate τ_x on extraction
4. a rising tax rate $\tau_x(t)$ on extraction

Using simple mathematics and figures, I will show tax of type 1 has no effect on extraction, while taxes of type 2 and 3 have the same effect as an increase in extraction costs. A tax of type 4 could be justified as a climate policy, see Hoel and Kverndokk (2006). This tax type is discussed more in Hoel (2011), please read!