Seminar 5: Discounting & Sustainability

November 17, 2011

Exercise

Part 1 Suppose a father inherits a family fortune of 10 million NOK and he wants to use some of it for himself (to be precise, the share x_1) but also to bequest some of it to his child (the share x_2) and grandchild (the share x_3). He does not distinguish between his offspring but is somewhat impatient himself, so that he assigns a share of 40% for himself and a share of 30% to his son and grandchild respectively. He thinks about the problem in the following way:

$$\max_{x_1,x_2,x_3} U = \ln x_1 + \beta(\ln x_2 + \ln x_3)$$

(1) Can you figure out which discount factor he applies?

(2) When the father has left this earth and it is the son's turn to dispose of the family fortune, does he follow his father's will (assume that he his as impatient as his father)? If not, how does the son share the remaining money?

(3) Now suppose that the father is aware of the son's lack of commitment. Could the father devise a sharing scheme which he is sure will be followed? How would such a scheme – if it exists – relate to the original plan (1) and the distribution in (2)?

Part 2 Consider an economy with an aggregate production function

$$Q_t = K_t^{\alpha}(M_t R_t)^{1-\alpha}$$

where K is capital, R is the use of an exhaustible resource (which can be extracted without costs) and M is a variable representing the technology level of the economy. All these variables depend on time t. $\alpha \in (0, 1)$.

(1) Show that the interest rate in the economy, i.e. the marginal productivity of capital, is constant provided $\frac{MR}{K}$ is constant.

(2) Show that when the interest rate is constant, the resource price, i.e. the marginal productivity of the resource, is proportional to M.

(3) Explain what is meant by intertemporal efficiency.

(4) Assume that M_t grows at a constant rate $m > 0$. Show that in this case it is possible to have an intertemporally efficient growth path along which the interest rate is constant and output grows at a constant rate g.

(5) What is the interest rate along the path described in (4)?

(6) What is the relationship between the growth rate g and the saving rate $s = \frac{\dot{K}}{Q}$ *along the path described in (4)?*

(7) Assume that the population is constant and that social welfare is given by the function

$$\int_0^\infty u(C_t)e^{-\rho t}dt$$

where $C = Q - \dot{K}$*. Derive the conditions for the optimal outcome, and show under what conditions a growth path of the type described in (4) is optimal (assume that* $\frac{-u''C}{u'}$ *is constant). When such a growth path exists, how does the growth rate depend on the parameters in the welfare function?*

Article

Hepburn, C.; Duncan, S. & Papachristodoulou, A. (2010): Behavioural Economics, Hyperbolic Discounting and Environmental Policy, *Environmental and Resource Economics*, 46, 189-206.

Link: `http://www.springerlink.com/content/504181j735048642/`