Fossil Fuels and Climate Change (Lecture 10)

This lecture is based on Hoel and Kverndokk (1996) and Hoel and Sterner (2007). Additionally, the paper from Heal (2009) is an excellent review of the issues at stake.

1 Climate change

1.1 The optimal carbon tax in an aggregated model

Setup of model follows Hoel and Kverndokk (1996, section 2), where $u(x)$ is the utility derived from consumption of the fossil fuel (assume $u'(0) = B < \infty$), $c(A)$ denotes the cost due to accumulated extraction A and $D(P)$ is the damage from the accumulated stock of carbon in the atmosphere. The discount rate is constant and given by r and the decay rate of carbon in the atmosphere is constant given by α. The problem of the social planner is thus

$$\begin{aligned} &\max_{x_t} \int_0^\infty e^{-rt} \left[u(x) - c(A)x - D(P)\right] \mathrm{d}t \\ &\text{subject to: } \dot{A} = x, \quad \dot{P} = x - \alpha P, \quad x \geq 0, \quad S_0 \geq \textstyle\int x \mathrm{d}t \end{aligned} \tag{1}$$

To solve this, we set up the current-value Hamiltonian:

$$\mathcal{H} = u(x) - c(A)x - D(P) + \lambda x + \mu(x - \alpha P) \tag{2}$$

and then we derive the corresponding necessary conditions for an optimum:

$$\frac{\partial \mathcal{H}}{\partial x} = u'(x) - c(A) + \lambda + \mu \leq 0 \quad (= 0 \text{ for } x_t > 0) \tag{3}$$

$$\dot{\lambda} - r\lambda = -\frac{\partial \mathcal{H}}{\partial A} = c'(A)x \tag{4}$$

$$\dot{\mu} - r\mu = -\frac{\partial \mathcal{H}}{\partial P} = D'(P) + \mu\alpha \tag{5}$$

$$\lim_{t\to\infty} e^{-rt}\lambda_t = 0 \tag{6}$$

$$\lim_{t\to\infty} e^{-rt}\mu_t = 0 \tag{7}$$

No regulation Consider the case where extraction is costless up to some limit $\bar{A}$ and then jumps to some level above B when $A_t > \bar{A}$. We are now in the very standard resource extraction case. Suppose that firms do not care about pollution. Without regulation, the consumer price $p = u'(x)$ equals the scarcity rent π (defined as the negative of λ) and both rise at the rate of interest. The fossil fuel stock will be emptied at some time T, defined by $\lambda_T = u'(0) = B$. The entire stock of fossil fuel $\bar{A}$ ends up as pollution in the atmosphere, although – when $\alpha > 0$ – some of it decays along the way and eventually all of it will decay once production has ended.

Optimal regulation A social planner who cares about pollution would install a carbon tax $\theta_t = -\mu_t$. The optimal tax is derived by solving the differential equation that corresponds to (5):

$$\dot{\theta} = (r + \alpha)\theta - D'(P) \quad \Leftrightarrow \quad \theta = K_0 e^{(r+\alpha)t} + \int_t^\infty D'(P)e^{-(r+\alpha)(\tau-t)}\mathrm{d}\tau \tag{8}$$

Due to the transversality condition (7) we know that $K_0 = 0$. The tax is thus corrects for the discounted future negative externalities due to accumulation of carbon in the atmosphere. When $\alpha = 0$ the future damages are only attenuated by discounting. The scarcity rent is not influenced by the tax, but the consumer price will be. Hence production will end later. When $\alpha > 0$, the pollution stock will fall whenever $x < \alpha S$. Since $S \to 0$ also $\theta \to 0$. The resource will be exhausted for sure. When $\alpha = 0$ the pollution stock will never fall and some of the resource may remain in the ground, namely if $D'(\bar{A}) > B$.

2 Discounting

2.1 Derivation of the "Ramsey rule"

Consider the following Ramsey model, where consumption $C(t)$ is a composite good yielding utility according to the time invariant function u, which is discounted at the rate δ:

$$U(C_t) = \int_0^\infty u(C_t)e^{-\delta t}\,\mathrm{d}t. \tag{9}$$

Abstract from population growth and technological progress in the following. The economy is such that capital K_t yields output $f(K_t)$ which can be devoted to investment or consumption according to the intertemporal constraint:

$$\dot{K}_t = f(K_t) - C_t. \tag{10}$$

Maximizing (9) subject to (10) involves the following Hamiltonian:

$$\mathcal{H} = u(C_t) + \mu[f(K_t) - C_t] \tag{11}$$

and includes the first-order-conditions:

$$u'(C_t) - \mu \leq 0 \quad (= 0 \text{ for } C > 0), \tag{12}$$

$$\dot{\mu} = \mu\delta - \mu f'(K_t). \tag{13}$$

Now differentiate (12) with respect to t: $\dot{\mu} = u''(C_t)\dot{C}_t$ and insert both $\dot{\mu}$ and μ into (13) to obtain the Euler equation:

$$u''(C_t)\dot{C}_t = u'(C_t)\delta - u'(C_t)f'(K_t). \tag{14}$$

Upon noticing that in an efficient market economy, the marginal productivity of capital $f'(K(t))$ equals the interest rate equals the consumption discount rate r we have:

$$r = \delta - \frac{\frac{\mathrm{d}}{\mathrm{d}t}u'(C_t)}{u'(C_t)} \tag{15}$$

Further, denote the proportional growth rate of consumption by $g = \frac{\dot{C}(t)}{C(t)}$ and the elasticity of the marginal utility of consumption as $\eta = -\frac{u''(C(t))C(t)}{u'(C(t))}$, we see that (14) can be written as Ramsey rule:

$$r = \delta + \eta g \tag{16}$$

2.2 Discount rates for several consumption goods (Hoel and Sterner, 2007)

Now consider the case the when the natural resource is an argument in the utility function $U(C,R)$. The objective function is then:

$$U(C_t, R_t) = \int_0^\infty u(C_t, R_t) e^{-\delta t} \, \mathsf{d}t. \tag{17}$$

which, parallel to (15), leads to:

$$r = \delta - \frac{\frac{\mathrm{d}}{\mathrm{d}t} u'_C(C_t, R_t)}{u'_C(C_t, R_t)} \tag{18}$$

The valuation of the natural resource is given by $\frac{U_R}{U_C}$, telling which increase in current consumption is necessary so that we can accept the deterioration of the natural resource by one unit. The relative change in price is therefore:

$$q = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{U_R}{U_C}\right)}{\left(\frac{U_R}{U_C}\right)} \tag{19}$$

The combined effect on the evaluation of future projects $d = r - q$ is ambiguous without further specification of the utility function. Therefore, consider the Cobb-Douglas function:

$$U(C,R) = C^{1-\gamma} R^\gamma \tag{20}$$

Then from (18) and (20) it can be derived that[1]

$$r = \delta + \gamma (g_C - g_R) \tag{21}$$

where g_C denotes the consumption growth rate and g_R the environmental growth rate. For the relative price of the natural resource, it finally follows from (19) and (20) that:[2]

$$q = g_C - g_R \tag{22}$$

In the most reasonable case that $g_C > g_R$, the resulting interest rate $d = r - q$ is significantly lower than the conventional discount rate (it might even be negative). Taking change of relative prices induced by the scarcity of the natural resource into account therefore leads to a discounting scheme which is much more in line with common concern that the standard cookbook economic analysis with exponential discounting gives insufficient weight to future environmental problems.

[1] The derivation is straightforward, but a little tedious:

$$\begin{aligned}
\frac{\frac{\mathrm{d}}{\mathrm{d}t} u_C(C)}{u_C(C)} = \frac{u_{CC}\dot{C} + u_{CR}\dot{R}}{u_C} \quad &= \frac{u_{CC}C}{u_C}\frac{\dot{C}}{C} + \frac{u_{CR}R}{u_C}\frac{\dot{R}}{R} \\
&= \frac{-\gamma(1-\gamma)C^{-\gamma-1}R^\gamma C}{u_C}\frac{\dot{C}}{C} + \frac{\gamma(1-\gamma)C^{-\gamma}R^{\gamma-1}R}{u_C}\frac{\dot{R}}{R} \\
&= -\gamma\left(\frac{\dot{C}}{C} - \frac{\dot{R}}{R}\right)
\end{aligned}$$

[2] Again, the derivation is an example of fun and games with Cobb-Douglas:

$$\frac{U_R}{U_C} = \frac{\gamma}{1-\gamma}\frac{C}{R}; \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{U_R}{U_C}\right) = \frac{\gamma}{1-\gamma}\frac{\dot{C}R - C\dot{R}}{R^2}$$

$$q = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{U_R}{U_C}\right)}{\left(\frac{U_R}{U_C}\right)} = \frac{\dot{C}}{C} - \frac{\dot{R}}{R}$$

2.3 Declining discount rates (hyperbolic discounting)

Discount rates could be (modeled as) declining because:

- known changes in the growth rate
- increasing marginal willingness to pay for the environment
- uncertainty about the discount or the growth rate
- observed behavior

How uncertainty about the correct discount rate implies a declining discount rate: Let $\delta_1 < \delta_2$ and let the chance that δ_1 is the correct rate be given by p. The discount factor to apply to some future date t is

$$pe^{-\delta_1 t} + (1-p)e^{-\delta_2 t}$$

The (absolute) rate of change of this discount factor, the instantaneous discount rate, is:

$$w_1\delta_1 + w_2\delta_2$$

where $w_1 = pe^{-\delta_1 t}$ and $w_2 = (1-p)e^{-\delta_2 t}$. We have:

$$\lim_{t\to\infty} \frac{w_2}{w_1} = \lim_{t\to\infty} \frac{1-p}{p} e^{(\delta_1-\delta_2)t} = 0 \quad \Rightarrow \quad \lim_{t\to\infty} (w_1\delta_1 + w_2\delta_2) = \delta_1$$

References

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