Lecture 7: Optimal management of renewable resources

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Overview

This lecture note gives a short introduction to the optimal management of renewable resource economics. A text book treatment can be found in Perman et al. (2003, ch. 17).

1 Introduction

The economics of renewable resource use is essentially a multi-disciplinary undertaking, integrating both biological and economic aspects. When modeling the dynamics of the resource, one has to choose a level of analysis. An intuitive entity is the organism itself (a fish, a tree, or a cow) that experiences growth and mortality. While growth and mortality determine the dynamics of the existing number of individuals, it is the potential to reproduce which characterizes *renewable* resources. Resources whose reproduction is completely outside of the control of the resource users can perhaps best be analyzed in the framework of "eating a cake of unknown size". Aquaculture could be an example of such a resource system. For most resource management problems however, it will be useful to model a reproduction function which depends in some (possibly highly nonlinear, possibly very stochastic) way on the existing number of individuals, which in turn are influenced by the current exploitation regime.

In addition to the viewpoint of an organism, one could also focus on the dynamics of the underlying processes (e.g. taking the gene as basic observation when discussing harvest-induced evolution, or taking an energy unit as basic observation when analyzing food-web models). Most models will take an aggregated view, i.e. analyzing a fishery, a forest, an ecosystem as a whole. This is what we do in the following. The biomass of the resource stock will be denoted by S and its growth function by $G(S)$. Denote the harvest by H. The stock dynamics of the renewable resource economy are then given by: $\dot{S} = G(S) - H$.

The most commonly used growth function is probably the "logistic model":

$$G(S) = gS\left(1 - \frac{S}{S_{max}}\right) \tag{1}$$

It captures the idea that there is a maximum stock size S_{max}, determined by the carrying capacity of the ecosystem, so that $G(S_{max}) = 0$. Similarly, no fish fall from heaven (i.e. $G(0) = 0$). In between $S = 0$ and $S = S_{max}$, growth $G(S)$ will be positive and the resource stock will grow with the "intrinsic growth rate" g at the origin and it will grow strongest at $\frac{1}{2}S_{max}$. On the one hand, the logistic growth function allows to describe the most important features of renewable resources in a very simple and plain manner. On the other hand, the logistic growth function does not allow to analyze the internal dynamics of the stock (e.g. its age structure) and issues such as a minimum viable population size.

The latter can be introduced by allowing for *depensation*: This means that there exist some values of S for which the "per-capita" growth rate of the stock is increasing. *Critical depensation* then means that there exist some minimum viable population level S_{min} below which the per-capita growth rate of the stock is negative. In contrast, the standard growth function (1) exhibits *compensation*, i.e. the per-capita growth rate of the stock is decreasing over its entire domain.

The harvest function

Harvest is commonly thought to depend on the amount of effort E that is applied to catch the fish and on the size of the fish stock S itself. Perhaps *the* canonical harvest function is

$$H(E, S) = qES, \tag{2}$$

Where harvest is proportional to effort and q is called the "catchability-coefficient", translating one unit of effort (e.g. measured in number of vessels) into one unit of harvest (mostly measured in kg). Together with the logistic-growth function, it is referred to as Gordon-Schaefer model after the seminal contributions of Gordon and Schaefer (both published in 1954).[1] Note that this harvest function is a special form of the Cobb-Douglas production function (which is very common in economics) where both $\alpha = 0$ and $\beta = 1$.

$$H(E, S) = qE^{1-\alpha}S^{\beta}. \tag{3}$$

First, $1-\alpha$ is the effort-output elasticity, specifying by how much, all else remaining the same, the harvest increases when effort increases by one unit. Most often harvest is thought to be proportional to the amount of effort used and α is set to 0. Decreasing returns ($\alpha > 0$) could be thought of as representing a crowding externality.[2] Second, β denotes the stock-output elasticity. The value of β tells how much, all else remaining the same,, the harvest increases when the stock increases by one unit. If the fish stock follows an ideal free distribution ($\beta = 1$) always occupying a given area, the density of fish declines at the same rate as the stock gets depleted. Contrarily, for a perfect schooling species ($\beta = 0$), the fish density

[1] Gordon (1954) was the first to make the malign effect of missing property rights in fisheries known to a wide public. However, largely unnoticed, the Danish economist Warming has described these mechanisms already in 1911 (translated by Andersen, 1983).

[2] An *externality* is "a cost or benefit that results from an activity or transaction and that affects an otherwise uninvolved party who did not choose to incur that cost or benefit" (wikipedia). A *crowding externality* then means for example that each fishermen's use of space inhibits the other fishermen in their free us of space.

remains unchanged, but the area occupied by the fish declines as the stock gets depleted. In this case, the harvest function is independent of the stock size! Note that for $\beta > 0$, the stock dependency makes it excessively costly to harvest the last fish in the ocean. This can be more clearly seen by an inspection of the cost function. Suppose that harvesting costs are proportional to effort: $c(E) = w \cdot E$ and use equation (3) with $0 < \alpha, \beta < 1$ to re-write the cost function in terms of harvest H and stock size S:

$$\begin{aligned} H &= qE^{1-\alpha}S^{\beta} \\ E^{1-\alpha} &= \frac{H}{qS^{\beta}} \\ E(H,S) &= \left(\frac{H}{qS^{\beta}}\right)^{\frac{1}{1-\alpha}} \\ c(H,S) &= wE = w\left(\frac{H}{qS^{\beta}}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{4}$$

Under these assumptions, costs are decreasing in S and increasing in H.

This re-formulation will be useful in the following, as it allows us to harvest H directly as a "choice" or "control" variable. Later (in section **??**), we will again use effort E as our choice variable.

2 Rent maximization

Consider again the dynamics of the renewable resource:

$$\dot{S} = G(S) - H \tag{5}$$

where $G(S)$ is a concave function with a unique maximum. Clearly, there is an indefinite number of equilibria in this system. In other words, all harvest levels that take out as much as regrows are sustainable. The question thus is which sustainable harvest level is the optimum?

Stock managers often call for "maximum sustainable yield" (MSY). Obviously, the objective of harvesting as much as can maximally be sustained places no concern whatsoever on economic criteria. Some have therefore called for "maximum economic yield" (MEY), often interpreted as the sustainable stock size which gives the highest surplus (calculated as revenue over cost). Perman et al. (2003, p.573) call this the "static private-property steady state". It is static because the objective of maximizing equilibrium profits neglects both dynamics and discounting.

Let us then consider the "correct" control-theoretic problem of optimal harvesting. Assume there is a sole-owner of a fishery whose dynamics are given by (5). Her objective is to maximize the net-present value of this fishery, i.e. the discounted (at the rate δ) sum of revenues from harvesting (pH) minus the cost of harvesting ($c(H,S)$, with the usual properties $c_H > 0, c_{HH} \geq 0, c_S < 0, c_{SS} \leq 0$).[3]

$$\begin{aligned} &\max_H \int_0^\infty [pH - c(H,S)]\, e^{-\delta t} \mathrm{d}t \\ &\text{subject to:} \quad \dot{S} = G(S) - H; \quad S(0) = S_0, S \geq 0; \quad 0 \leq H \leq H_{max} \end{aligned} \tag{6}$$

The current value Hamiltonian is then:

$$\mathcal{H} = pH - c(H,S) + \mu \left(G(S) - H\right), \tag{7}$$

and necessary conditions for optimality include[4]

$$p - c_H(H,S) - \mu \leq 0 \quad (= 0 \text{ for } H > 0), \tag{8}$$

and the shadow price μ must obey:

$$\dot{\mu} = \mu \left(\delta - G'(S)\right) + c_S(H,S). \tag{9}$$

From equation (8) we see that the shadow price μ has an interpretation as resource rent, i.e. the net of price over marginal cost. Equation (9) can then be re-written as follows:

$$\delta = \frac{\dot{\mu}}{\mu} - \frac{c_S(H,S)}{\mu} + G'(S) \tag{9'}$$

This equation says that resource owner must be indifferent between taking out one unit of stock and investing its value in the financial market (left-hand-side of the equation) or leaving it in the ocean, where in addition to the proportionate growth in net price, it yields a proportionate reduction in harvesting cost (due to the marginal increase in stock size) and an increase in natural growth (the right-hand-side of the equation).

The optimal development of the renewable resource economy is to steer the stock from its initial value S_0 to the optimal equilibrium S^* and continue harvesting $H^* = G(S^*)$ forevermore. When the economy is described by the traditional Gordon-Schaefer function (2), we see that the Hamiltonian (7) is linear in the control H yielding the bang/bang solution $H^* = 0$ if $S < S^*$; $H^* = H_{max}$ if $S > S^*$. The approach dynamics are then characterized by

[3]Where the notation c_H is a short-hand for "the derivative of the function c with respect to the variable H and c_{HH} accordingly denotes the second derivative with respect to H.

[4]For the full set of sufficient conditions see e.g. Sydsæter et al. (2005): *Further Mathematics*, p.348.

the "most rapid approach path" (MRAP).

A characterization of the harvest level $H^* = H(S)$

Let us give a further characterization of the optimal harvest level, and consider harvesting costs of the form $c(H,S) = w\left(H/qS^\beta\right)^{\frac{1}{1-\alpha}}$, for $\alpha \in (0,1)$ (see equation 4). From the (economic) first-order condition (8) it follows that if harvesting is profitable at all, the optimal harvest H^* can be expressed as a function of stock size:

$$\begin{aligned}
c_H(H^*,S) \;&=\; \frac{\partial\left[w\left(H/qS^\beta\right)^{\frac{1}{1-\alpha}}\right]}{\partial H} = p-\mu \\
\frac{1}{1-\alpha} w H^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{qS^\beta}\right)^{\frac{1}{1-\alpha}} &= p-\mu \\
H^{\frac{\alpha}{1-\alpha}}\left(\frac{1}{qS^\beta}\right)^{\frac{1}{1-\alpha}} &= \frac{(p-\mu)(1-\alpha)}{w} \\
H^{\frac{\alpha}{1-\alpha}} &= \frac{(p-\mu)(1-\alpha)}{w}\left(qS^\beta\right)^{\frac{1}{1-\alpha}} \\
H^* &= \underbrace{\left(\frac{(p-\mu^*)(1-\alpha)}{w}\right)^{\frac{1-\alpha}{\alpha}} q^{\frac{1}{\alpha}}}_{A} S^{\frac{\beta}{\alpha}} \quad = \quad A S^{\frac{\beta}{\alpha}}
\end{aligned} \qquad (10)$$

The (biological) equilibrium condition $\dot{S} = 0 = H(S^*) - G(S^*)$ then allows to draw both growth and harvest function in one diagram to read off the steady state stock value S^*.

Comparative statics

To gain some further insight, let us look at the situation once steady state has been reached, i.e. a situation where $S = S^*$ and $H = G(S)$. There are basically three measures of interest: effort, harvest, and stock. How their values change with the exogenous parameters p, q, and w will often depend on the specific model specification.

A formal investigation for the stock size S^* would require to differentiate the equilibrium condition $0 = H(S) - G(S)$ with respect to the parameter of interest e.g.:

$$0 = \frac{\partial H(S)}{\partial S}\frac{\partial S}{\partial p} + \frac{\partial H(S)}{\partial p} - \frac{\partial G(S)}{\partial S}\frac{\partial S}{\partial p}$$

Which – provided that $G'(S) < H'(S)$ – can be solved for:

$$\frac{\partial S}{\partial p} = \frac{\frac{\partial H(S)}{\partial p}}{\frac{\partial G(S)}{\partial S} - \frac{\partial H(S)}{\partial S}}$$

Inferring the direction of change thus boils down to analyzing how A in equation (10) changes with prices, technology, and costs. It is then relatively straightforward to see that the equilibrium stock will increase with the resource price (a higher p) and technology (a higher q), and it will decrease when marginal effort costs w are higher.

	p	q	w
Stock	-	-	+
Harvest	?	?	?
Effort	?	?	?

Table 1: Comparative Statics for Optimal Steady State

Whereas the direction of change is unequivocal for the steady state stock S (see row 1 in Table 1), this is not the case for steady state harvest and effort (row 2 and 3 in Table 1). The direction of the effects can unambiguously stated when S is larger (smaller) than S_{MSY} both before and after the change. For example, when $S^* > S_{MSY}$ and $S^{*new} > S_{MSY}$ (so both old and new steady state stock are to the right of the peak of $G(S)$), then an increase in price leads to a higher harvest and higher effort. Whether the steady state stock is larger or smaller than S_{MSY} depends on whether:

$$G'(S^*) \overset{?}{<} \delta + \frac{c_S(H^*, S^*)}{p - c_H(H^*, S^*)}$$

The role of discount rate δ has not been discussed so far. Due to equilibrium, we have $\dot{\mu} = 0$, and $\mu(t) = \mu^* = p - c_H(H^*, S^*)$, and from (9') it follows:

$$\mu^* = \frac{c_S(H^*, S^*)}{G'(S^*) - \delta} \tag{11}$$

Recall that $c_S(H^*, S^*) < 0$ and $G'(S^*) < \delta$, so that the optimal shadow value – the resource rent – is the larger, the smaller is δ. When $\delta = 0$, (11) can be written as:

$$\mu^* G'(S^*) = c_S(H^*, S^*) \tag{12}$$

stating that profits are maximized when the marginal revenue (with respect to stock changes) equals the marginal cost (with respect to stock changes), which is exactly the "static private-property steady state" mentioned above. For $\delta > 0$, (11) can be written in present-value terms:[5]

$$p - c_H(H^*, S^*) = \frac{\Pi'(S^*)}{\delta} \tag{13}$$

where the left-hand-side is the present value of catching one more unit of the stock today while the right-hand-side give the cost in terms of the present value of the reduced permanent income due to the changed steady state. Now as the discount rate increases, the future income from the stock has less and less present value. In the limit, as $\delta \to \infty$, the right-hand-side tends to zero and the optimal action is to deplete the resource until the point where marginal revenue just equals marginal price, hence no profits are made.

References

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Perman, R., Common, M., McGilvray, J., and Ma, Y. (2003). *Natural Resource and Environmental Economics.* Pearson Education, Harlow, 3rd edition, ch. 17.

[5] Where we have made use of $G(S^*) = H^*$, $\mu^* = p - c_H(H^*, S^*)$, and consequently write current profits as: $\Pi(S^*) = pG(S^*) - c(G(S^*), S^*)$. Hence (11) can be written as: $r\mu^* = \mu^* G'(S^*) - c_S(H^*, S^*) \Rightarrow \delta\,(p - c_H(H^*, S^*)) = pG'(S^*) - c_H(G(S^*), S^*)G'(S^*) - c_S(G(S^*), S^*) \Rightarrow \delta\,(p - c_H(H^*, S^*)) = pG'(S^*) - c_H(G(S^*), S^*)G'(S^*) - c_S(G(S^*), S^*)$.

Table 2: Definition (and some explanation) of variables used in the text

Variable	Meaning
S	Stock size
$G(S)$	Stock growth as a function of S
H	Harvest
$\dot{S}$	Change of stock over time, shorthand for $\frac{\mathrm{d}S}{\mathrm{d}t}$
g	Growth rate of the stock when $S \to 0$
S_{max}	Maximum stock size (carrying capacity)
S_{min}	Minimum viable stock size
E	Effort
q	Catchability coefficient
$1-\alpha$	Effort-output elasticity: How much does output change when E changes by 1%?
β	Stock-output elasticity: How much does output change when S changes by 1%?
$c(...)$	Cost function
w	Cost per unit of effort
δ	Discount factor: captures how much we discount (value less) future consumption in relation to today's consumption (e.g. because we are impatient).
μ	Shadow price, a technical variable from the optimization problem. It tells by how much the maximized value of the objective function would increase when the constraint would be relaxed a tiny bit.