

## Resource Economics – Seminar 6

In this last seminar, you are given the opportunity to think about issues that we didn't cover in the previous seminars. Specifically, we will explore dynamic common-pool problem for non-renewable resources, renewable resources, and discounting for long term projects such as nuclear power, climate change, etc. It is important that you come prepared and attend this seminar. If you have difficulty in formulating the problems or get stuck in the early phases of the solutions, drop by my office (ES 1041).

**Exercise 1:** Consider two oil firms that have adjacent concessions for their reservoirs  $S_1$  and  $S_2$ . In reality, these two reservoirs form a common pool, so that the oil flows under ground from one reservoir to the other at a rate proportional to the difference between the two stocks:

$$\dot{S}_{it} = -x_{it} + \alpha(S_{jt} - S_{it}) \quad \text{for } i, j = 1, 2$$

where  $x_{it}$  is the extraction of player  $i$  at time  $t$ . Suppose that extraction is costless, that both firms have the same discount rate. Demand is given and there is capacity constraint for extraction i.e.  $x_{it} \leq \bar{x} \forall t$ . Each firm takes the extraction profile of the other firm as given and maximizes discounted revenues.

- (i) Derive the equilibrium extraction profile of the firms
- (ii) Discuss the dynamic common-pool problem based on your result in (i)
- (iii) Suppose an add-valorem tax  $\tau$  (so that the producer price  $p$  equals  $p = q(1 - \tau)$  where  $q$  is the consumer price) is levied by the authorities to alleviate the problem. Derive the optimal time profile of this tax and discuss
- (iv) What happens to extraction path as the primitives of the model change i.e find comparative static results with respect to  $r, \alpha, \bar{x}$  [Bonus question]

Finally, write a summary about what you have learnt from this problem.

**Exercise 2:** Suppose that there are two countries that derive logarithm utility from consuming fish. They share the same fish stock, which evolves according to the following dynamics:

$$x_{t+1} = \left( x_t - c_t^i - c_t^j \right)^\alpha$$

where  $\alpha \in (0, 1)$ . Suppose that there are only two time periods ("now" and "the future"), suppose that both countries discount the future with the same factor  $\beta \in (0, 1)$ , and suppose that whatever is the second period fish stock is shared equitably. Characterize the non-cooperative Nash equilibrium and contrast it to the socially optimal solution. In addition, write a summary about what you have learnt from this problem.

**Exercise 3:** The last exercise is based on two important papers about discount rates for long term investment decisions. The first is *Weitzman Martin L., 1998. "Why the Far-Distant Future Should Be Discounted at Its Lowest Possible Rate" Journal of Environmental Economics and Management, Volume 36, Issue 3, November 1998, Pages 201 - 208.* The second paper is *Martin L. Weitzman, 2001. "Gamma Discounting" American Economic Review, American Economic Association, vol. 91(1), pages 260-271, March.* We start with Weitzman's 1998 paper and the proceed to the 2001 AER paper. You don't need to read Weitzman (1999) but please read Weitzman(2001) if you can.

By now you have noted that that no one knows the correct rate for discounting the distant future. Instead, there are various discount rates with various probabilities of being the correct discount rate. The fact of the matter is that we need to decide about mitigation of climate change *now*, and that just after we make the decision will the actual discount rate be realized according to the above probability distribution in future. We want to discount the value of a future cost or benefit at time  $t$  back to time zero, before we know what will be the actual realized discount rate. Let the effective interest rate for this purpose be  $r(t)$ .

- (i) To have a quick stab at the problem, let's focus on Weitzman(1994) i.e. suppose there are  $n$  possible discount rates and that discount rate value  $r_i$  has probability  $p_i$  of turning out to be the right discount rate to use. Give some kind of an argument why  $r(t)$  should satisfy the condition  $e^{-r(t)t} = \sum_{i=1}^n p_i e^{-r_i t}$ .
- (ii) Show that (a)  $r(0) = \sum_{i=1}^n p_i r_i$ , (b)  $\lim_{t \rightarrow \infty} r(t) = \min\{r_i\}$ , and (c) provide economic intuition why this is the case.
- (iii) Show [atleast argue] that  $\frac{d}{dt}r(t) < 0$
- (iv) Now let's confront the problem in Weitzman(2001). To do so, assume the discount rates are generated from the Gamma distribution i.e.  $f(r_i) = \frac{\beta^\alpha}{\Gamma(\alpha)} r_i^{\alpha-1} e^{-\beta r_i}$  and  $r_i \in [0, \infty)$ . Give some kind of an argument why  $r(t)$  should satisfy the condition  $e^{-r(t)t} = \int_0^\infty e^{-r_i t} f(r_i) dr_i$ .
- (v) Show that (a)  $e^{-r(t)t} = (\frac{\beta}{\beta+t})^\alpha$ , (b) that  $\frac{d}{dt}r(t) < 0$ , and argue that long term investment projects need to be discounted at a decreasing rate.

Finally, write a summary about what you have learnt from this problem as well; and email the summaries of the three problems to your best at t.k.mideksa@econ.uio.no before Tuesday evening.