

Seminar 4 - exhaustible resources

Exercise 1: Investment and extraction under expropriation after one period

This is a simplification of the model in Bohn & Deacon (2000). Consider the following problem. In period 0 the owner of a resource decides how much A to invest in extraction technology which lowers the extraction costs. The cost of making these investment is A^2 . In period 1 the owner decides how much to extract (E_1) given world prices p (exogenous) and costs of extraction $C = E_1^\theta/A$. The stock is infinite. Parameters are such that $\theta > 3/2$ and $\theta > p > 1$.

1. Set up the problem of the owner in choosing A and E_1 .
2. Solve the problem with backward induction. What is the optimal E_1 and A ?

Exercise 2: Investment and extraction under expropriation after two periods

We continue with a similar problem but with two periods under which the owner can extract. In period 0 the owner of a resource decides how much A to invest in extraction technology which lowers the extraction costs. The cost of making these investment is A^2 . In period 1 the owner decides how much to E_1 extract given world prices p (exogenous) and immediate costs of extraction $C = E_1^\theta/A$. In period 2 the owner decides how much to extract given the same price but with extraction costs $C = cE_1 + E_2^\theta/A$. There is no discounting and the stock is infinite. Parameters are as in exercise 1 but with the addition $c < p$.

1. Set up the problem of the owner in choosing A and E_1 and E_2 .
2. Solve the problem with backward induction. What is the optimal E_1 , E_2 and A ?
3. How do these values of extraction and investment compare to the values from exercise 1? Why?
4. More generally, discuss what the effect is on investment and extraction when one increases the time to expropriation (see Bohn & Deacon, 2000).
5. Suppose that the government which expropriates gets both the resource itself (from which it is planning to extract) and the capital

investments (A) that the original owner has made. Discuss which aspects this government should think about when deciding which contract length to give to the original owner.

Exercise 3: Exploration investment under taxation

1. Royalties: A firm is exploring for resources (S) in a single period. Every unit of S is worth p but the firm has to pay a royalty tax τ on its revenues. To find S units the firm incurs costs of exploration $C = S^\theta$. a) Solve the exploration problem of the firm, how much does it explore? What is the effect of royalties on exploration?. b) The government sets the royalty rate τ to maximize its total tax revenues. What is the optimal tax from the government's point of view?
2. Profits tax: Suppose the tax is based on profits instead. a) Solve the problem of the exploration firm, how is exploration affected by the tax? b) What is the optimal tax from the government's point of view?
3. Uncertainty: Suppose now that if the firm takes costs C then with probability π it finds $S = C^{1/\theta}$ of the resource and with probability $1 - \pi$ it finds nothing. What is the effect of a profit tax only on firms which make a finding? Suppose the government can, as a complement to the profit tax, subsidize firms who make a loss. Show that such a system is non-distortionary.