ECON4925 Resource economics, Autumn 2015

Lecture 2: Renewable resources (fish)

Reading list:
Perman et al., Ch. 17

Outline of lecture:
(with reference to most central sections in Perman)

1. Biological growth processes. Introduction and sec. 17.1
2. Harvesting costs. Sec 17.3.1.2.1 and sec. 17.3.1.2.2
3. Socially optimal harvesting. Sec. 17.8.3, 17.8.4, and 17.11 to 17.11.3
4. An open access fishery. Sec. 17.3
5. Efficient and inefficient regulation. Sec. 17.14
   - Landing tax
   - Tradable quotas
   - Limits on total catch or duration of harvesting season

Socially optimal harvesting

Omitting time references, let x denote harvest and let S denote the resource stock. Costs are given by $C(x, S)$ with $C_x > 0$ and $C_S < 0$. As a special case we consider $C(x, S) = xc(S)$ with $c' < 0$.The output price is exogenous and constant equal to p. (Alternatively, one could assume that gross benefits of the harvest are given by $u(x)$ instead of px, so that p is replaced by $u'(x)$).

Optimal harvest is given by

$$Max \int_0^\infty e^{-rt} \left[px - C(x, S) \right] dt$$

subject to

$$\begin{aligned} \dot{S} &= G(S) - x \\ 0 &\leq x(t) \leq \bar{x} \\ S(t) &\geq 0 \end{aligned}$$

The current value Hamiltonian is :

$$H = px - C(x, S) + \lambda \left[G(S) - x \right]$$

Note: From the interpretation of $\lambda(t)$ it follows that $\lambda(t) > 0$.
Conditions for optimum (omitting time references):

$$\begin{aligned} \dot{\lambda} - r\lambda &= -\frac{\partial H}{\partial S} = C_S - \lambda G' \\ \frac{\partial H}{\partial x} &= p - C_x - \lambda = 0 \; (for \; 0 < x < \bar{x}) \\ Lim_{t \to \infty} e^{-rt} \lambda(t) S(t) &= 0 \end{aligned}$$

Assume that the optimal solution approaches (asymptotically) a stationary state, i.e. $\dot{\lambda} = \dot{S} = 0$. In this case it follows from the equations above that the stationary state (λ^*, S^*, x^*) is given by

$$\begin{aligned} x^* &= G(S^*) \\ C_x(x^*, S^*) + \lambda^* &= p \\ \lambda^* &= \frac{-C_S(x^*, S^*)}{r - G'(S^*)} \end{aligned} \tag{1}$$

For the special case of $C(x, S) = xc(S)$ this may be written as

$$\begin{aligned} x^* &= G(S^*) \\ c(S^*) + \lambda^* &= p \\ \lambda^* &= \frac{-x^* c'(S^*)}{r - G'(S^*)} \end{aligned} \tag{2}$$

Using a phase diagram, we can show that this stationary state is approached by $x(t) = 0$ for $S(t) < S^*$ and $x(t) = \bar{x}$ for $S(t) > S^*$.

Open access harvesting

Long-run equilibrium implies zero profit. In a stationary state we therefore have:

$$\begin{aligned} x^0 &= G(S^0) \\ px^0 &= C(x^0, S^0) \end{aligned} \tag{3}$$

For the special case of $C(x, S) = xc(S)$ this may be written as

$$\begin{aligned} x^0 &= G(S^0) \\ p &= c(S^0) \end{aligned} \tag{4}$$

Since $\lambda^* > 0$ we have $c(S^0) > c(S^*)$, implying $S^0 < S^*$.