ECON4925 Resource economics, Autumn 2015

Lecture 3: Forest resources

Updated August 31, 2015

Reading list:

Perman et al., Ch. 18

Holtsmark et. al

Outline of lecture:

(with reference to most central sections in Perman)

1. What are the issues? Introduction and sections 18.1 and 18.2
2. Optimal management of a commercial forestry. Sec 18.3 (in particular 18.3.2).
   - using the same model as for fish
   - using the Faustmann model
3. Other forest services. Sec. 18.4 and 18.5
   - Recreation services
   - Ecological services
   - Biodiversity
   - Carbon sink, see in particular Holtsmark at al and this lecture note
4. Alternative uses of land. Sec. 18.6

Forest resources: The optimal rotation period

Present value of forest with rotation period T:

$$V(T) = v(T) + e^{-rT}V(T)$$

where $v(T) = e^{-rT}m(T) - k$ (we have set the price of timber equal to 1). The expression above may be rewritten as

$$V(T;k,r) = \frac{e^{-rT}m(T) - k}{1 - e^{-eT}} = \frac{m(T) - e^{rT}k}{e^{rT} - 1} = \frac{m(T) - k}{e^{rT} - 1} - k$$

To find the optimal value of T we differentiate w.r.t. T:

$$V_T(T;k,r) = \frac{1}{\left(e^{rT}-1\right)^2}\left[\left(e^{rT}-1\right)m'(T) - \left(m(T)-k\right)re^{rT}\right] = 0$$

or

$$V_T(T;k,r) = \frac{1}{e^{rT}-1}\left[m'(T) - \frac{\left(m(T)-k\right)re^{rT}}{e^{rT}-1}\right] = 0$$

or

$$V_T(T;k,r) = \frac{1}{e^{rT}-1}\left[m'(T) - \frac{r}{1-e^{-rT}}\left(m(T)-k\right)\right] = 0$$

This may be rewritten in several ways. In particular:

$$m'(T) = r\frac{\left(m(T)-k\right)}{1-e^{-rT}} = r\left[m(T) + \frac{e^{-rT}m(T)-k}{1-e^{-rT}}\right] = r\left[m(T)+V(T)\right]$$

Notice that $V(T) > 0$ implies that the optimal T satisfies $\frac{m'(T)}{m(T)} > r$.

Let B denote the square brackets in V_T. The signs of V_{Tk} and V_{Tr} are equal to the signs of B_k and B_r. It can be shown that $B_k > 0$ and $B_r < 0$. We therefore have:

$$\frac{\partial T}{\partial k} = -\frac{V_{Tk}}{V_{TT}} > 0$$
$$\frac{\partial T}{\partial r} = -\frac{V_{Tr}}{V_{TT}} < 0$$

Forests as carbon sinks

This section shows some of the points made in Holtsmark et al., but using the approach of

Hoel, Michael and Thea Marcelia Sletten, "Wood-Based Bioenergy", CESifo Working Paper No. 4686, March 2014 | Abstract | PDF Download

We model the total volume of the forest in the same way as fish, with

$$\dot{S} = G(S) - x$$
$$0 \leq x(t) \leq \bar{x} \text{ where } \bar{x} \text{ is "large"}$$
$$S(t) \geq 0$$

The total stock will be higher the higher is the average age at harvesting, see appendix at the end of this lecture note.

Ignoring planting costs k, the optimal outcome is found by solving

$$Max \int_0^\infty e^{-rt} pxdt$$

We instead solve a slightly more general model. Assume that the flows of carbon into and out of the atmosphere are valued at q. Moreover, assume that when the forest is harvested, a fraction β of the carbon is immediately released into the atmosphere. The rest is stored in buildings etc for ever (see Holtsmark et al. for a more realistic case). The natural growth $G(S)$ of the forest absorbs carbon from the atmosphere. Given these assumptions, we solve

$$Max \int_0^\infty e^{-rt} \left[px + qG(S) - q\beta x\right] dt$$

The current value Hamiltonian is :

$$H = (p - q\beta)x + qG(S) + \lambda \left[G(S) - x\right]$$

Note: In this model it is not necessarily true that $\lambda(t) > 0$: If S is so large that $G' < 0$, a higher value of S will give a lower value of $qG(S)$, which cet. par. is "bad". If this outweighs the positive effect of a larger S making a higher x temporarily possible, we get $\lambda(t) < 0$.

Conditions for optimum (omitting time references):

$$\begin{aligned} \dot{\lambda} - r\lambda &= -\frac{\partial H}{\partial S} = -(q + \lambda)G' \\ \frac{\partial H}{\partial x} &= p - q\beta - \lambda = 0 \ (for\ 0 < x < \bar{x}) \\ Lim_{t\to\infty} e^{-rt}\lambda(t)S(t) &= 0 \end{aligned}$$

Assume that the optimal solution approaches (asymptotically) a stationary state, i.e. $\dot{\lambda} = \dot{S} = 0$. In this case it follows from the equations above that the stationary state (λ^*, S^*, x^*) is given by (provided we have an interior equilibrium; see below)

$$\begin{aligned} x^* &= G(S^*) \\ \lambda^* &= p - q\beta \\ \lambda^* &= \frac{qG'(S^*)}{r - G'(S^*)} \end{aligned} \tag{1}$$

Notice that for the special case of $q = 0$ we get the Faustmann Rule, which now takes the form $G'(S^*) = r$ (provided $p > 0$).

For $q > 0$ we have

$$\tilde{\lambda} \equiv \frac{\lambda^*}{q} = \frac{p}{q} - \beta = \frac{G'(S^*)}{r - G'(S^*)}$$

A stable steady state implies $G' < r$. It follows that an increase in $\tilde{\lambda}$ (for $p > 0$) must imply G' up , hence S^* down (since $G'' < 0$). The consequence of this increase of S^* on x^* depends on whether G' is positive or negative.

Notice that $\tilde{\lambda}$ increases if p increases or β declines. An increase in q will reduce $\tilde{\lambda}$ if $p > 0$ but increase $\tilde{\lambda}$ if $p > 0$.

In the reasoning above it was implicitly assumed that $S^* < \bar{S}$. If $p - q\beta - \lambda < 0$ we get $x = 0$, and the steady state will be given by $S^* = \bar{S}$. This will be the equilibrium outcome if

$$\frac{p}{q} - \beta < \frac{G'(\bar{S})}{r - G'(\bar{S})}$$

For q sufficiently small, this must hold if $p < 0$.

Appendix: A multi-vintage forest with constant rotation time

Consider a forest area consisting of N trees of varying vintages. A t year old tree has a size (measured in carbon content) $m(t)$, where $m(0) = 0$, $m(t) = \bar{m}$ for $t \geq \bar{T}$, $m'(t) > 0$ for $t \in (0, \bar{T})$, $m''(t) > 0$ for $t \in (0, \tilde{t})$ and $m''(t) < 0$ for $t \in (\tilde{t}, \bar{T})$.

Consider a constant rotation time of T years. This gives a steady state harvest equal to $h(T)$, and a steady state carbon stock equal to $S(T)$. If all trees grow till they are T years old, there are T vintages in the forest area, and N/T trees of each vintage. In particular, there are N/T trees of vintage T, the harvest is thus

$$h(T) = \frac{N}{T} m(T)$$

It is clear that $h'(T) > 0$ for $m'(T) > \frac{m(T)}{T}$ and $h'(T) < 0$ for $m'(T) < \frac{m(T)}{T}$. The rotation time that maximizes $h(T)$ is thus T^{MSY} defined by $m'(T^{MSY}) = \frac{m(T^{MSY})}{T^{MSY}}$. The value $h(T^{MSY})$ thus corresponds to $g(S^{MSY})$ in our setup.

The carbon content of each of the N/T trees of vintage t is $m(t)$. The carbon content of the whole forest is therefore

$$S(T) = \int_0^T \frac{N}{T} m(t)dt$$

It follows that

$$S'(T) = \frac{N}{T}\left[m(T) - \frac{1}{T}\int_0^T m(t)dt\right] > 0$$

Using l'Hospital's rule we find

$$Lim_{T\to 0} S(T) = Nm(0) = 0$$
$$Lim_{T\to\infty} S(T) = N\bar{m}$$

The latter value corresponds to $\bar{S}$ in our setup.

To conclude: Increasing T from 0 to T^{MSY} increases S from 0 to S^{MSY} and also increases the harvest from 0 to $g(S^{MSY})$. Increasing T further continues to increase S, reaching its limit $\bar{S}$ as T approaches infinity (i.e. no harvest). Increases in T beyond T^{MSY} give a monotonically declining harvest, reaching zero as T approaches infinity.