Resource Economics – Seminar 2

Thursday, September 17, 2015
Room HH201, 16:15-18:00

Problem 1 – Hotelling model

Consider the following model. A representative agent owns an exhaustible resource stock S_0 at the onset. There are no costs of extraction. Each resource owner solves the following problem:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t p_t E_t \\ \text{s.t. } S_{t+1} &= S_t - E_t \\ S_{t+1} &\geq 0 \end{aligned}$$

1. Solve the problem of the owner.
2. The resource is sold on a market to profit maximizing competitive firms who produce

   $$F(E) = AE^{\alpha}$$

   units of output where $\alpha < 1$.They sell their production for a price equal to 1. Solve the firms problem.
3. Define the general equilibrium price path.
4. Show that extraction is decreasing and price is increasing.
5. What is the extraction rate at time zero, i.e. E_0/S_0.
6. What is the extraction rate (E_t/S_t) in the other time periods. (Hint: solve the original problem as if it starts at time t).
7. Suppose there is an unexpected increase in the resource at time τ. What happens to the price and the extraction between $\tau - 1$ and τ?

Problem 2 – Hotelling with extraction costs

Consider the same model as previously but add an assumption that there are extraction costs $M(E)$ which are convexly increasing in E. Each resource owner then solves the following problem:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \left[p_t E_t - M(E_t) \right] \\ \text{s.t. } S_{t+1} &= S_t - E_t \\ S_{t+1} &\geq 0 \end{aligned}$$

1. Solve the problem of the owner.
2. The resource is sold on a market to profit maximizing competitive firms who produce

   $$F(E) = AE^{\alpha}$$

   units of output where $\alpha < 1$. They sell their production for a price equal to 1. Solve the firms problem.

3. Define the general equilibrium price path.
4. Given the answer in 3), what is the scarcity rent? How does it evolve over time?
5. Show that extraction is decreasing and price is increasing. (Hint: use an contradictive argument of the type: if $E_{t+1} > E_t$ then this contradicts the result in 3)
6. What would be the effect of an increase in β on the price path?
7. Suppose that the extraction costs are given by $M = E^{\theta}$. Suppose that the initial stock is very (=sufficiently) large, show that the profits/year follow a hill-shaped time path and that profits/unit/year are strictly increasing.

Problem 3 – Non-exhaustible resource

Consider the model by Farzin (1992) but in discrete time. A representative resource owner has a stock of infinite size but where unit extraction costs $M(Q)$ are increasing with cumulative extraction Q. Thus the owner solves:

$$\begin{aligned} & \max \sum_{t=0}^{\infty} \beta^t \left[p_t E_t - M(Q_t) E_t \right] \\ Q_t &= \sum_{\tau=0}^{t-1} E_\tau \\ M &= \gamma Q \end{aligned}$$

1. Solve the owner's problem.
2. The resource is sold on a market to profit maximizing competitive firms who produce

   $$F(E) = AE^{\alpha}$$

   units of output where $\alpha < 1$. They sell their production for a price equal to 1. Solve the firms problem.
3. What is the partial equilibrium extraction in one period?
4. How does extraction evolve in general equilibrium? Why? (Hint: show that if extraction is increasing this leads to a contradiction).
5. How does the resource price evolve over time?