JV; October 2016 – ECON 4925 (2016)

Lecture Note 2: Resource Extraction and Imperfect Competition[1]

This note presents some results related to resource extraction when we have a market structure between perfect competition and pure monopoly. Perfect competition has been the topic of several lectures, whereas monopoly extraction is covered in Stiglitz (1976) and the lecture on Sept 14. [2]

We focus first on a model with a dominant firm (a cartel, like OPEC) and a competitive fringe (independent resource owners, outside the cartel). We start with a static model so as to provide a simple representation of the "leader-follower"-notion, which highlights the importance of the sequence of moves by the players. (The natural equilibrium concept used here is the so-called Stackelberg equilibrium.) Thereafter we reverse the sequence of moves, and consider the resulting outcome as a Nash-Cournot equilibrium, and show the meaning of dynamic inconsistency within this setting. Then we turn to a dynamic representation of this dominant firm - competitive fringe model, but concentrate mainly on one solution concept - the so-called *open-loop Nash equilibrium* (a binding contract equilibrium or a pre-commitment equilibrium). In this game the dominant firm announces ex ante an extraction plan, while the members of the fringe, while pursuing their extraction plans, believe that the dominant firm will honour or stick to its announced plan. The strategies for all players will therefore be functions only of time and the resulting market outcome will constitute a Nash equilibrium. However, such an open-loop equilibrium will normally not be dynamically consistent, or constitute a subgame perfect Nash equilibrium. One should, by the way, bear in mind that there is an important distinction between subgame perfection and time or dynamic consistency, according to Karp and Newbery (1993; p. 891): "Perfect equilibria must be credible even if some agent deviates; time-consistent equilibria are defined without

---

[1] This note is covering a theme, "Oligopolistic Extraction", which has not been covered in class, but is meant as additional reading so as to see the role of mixed market structure. The other theme, "Dominant Firm – Competitive Fringe" was very briefly presented, but is not covered by the literature on the reading list. However, this issue, which is rather difficult, is so important that you should have some knowledge about it.

[2] Joseph Stiglitz (1976), Monopoly and the Rate of Extraction of Exhaustible Resources, *American Economic Review*, 66 (4), pp. 655 – 661.

regard to out-of-equilibrium behavor."[3] Because transactions are made between sovereign nations, "… contracts can only be enforced by mutual self-interest and not by appeal to higher authority…", to quote Newbery (p. 617) [4]. Hence, if the dominant firm should announce an extraction plan ("that seems to be the best one for the cartel"), then, if the fringe should believe that this plan will not be revised later, the dominant firm will normally have an incentive not to stick to the original plan, because it is possible to achieve an even higher profit from deviating from the original extraction plan. Because we assume that players are rational, the fringe will correctly predict that the dominant firm will deviate (at the same time the dominant firm is also able to predict the best strategy for the fringe and hence the outcome of the game), and the only outcome that no player will renege on, is the Nash equilibrium, not only for the entire game, but also for any conceivable subgame (or for any conceivable state that might happen). We should therefore look for a plan that is time consistent or involving a plan for any player that is best, and therefore credible, from any point in time, in the sense that it eliminates all empty threats. However, finding such (closed-loop) equilibria is normally very difficult because we have to leave the principle of optimality underlying optimal control theory which relies on the fact that future states or future values of the variables values do not affect current decisions.[5] At last we sketch a rather simple dynamic model for resource extraction under an oligopolistic (or duopolistic) market structure. This is a model where there is some "strategic interaction" through the impact each resource owner has on the market price. We describe the dynamic (open-loop) Nash Equilibrium under some rather restrictive assumptions.

Let us start with the introduction made by David Newbery, in a working paper from 1980, on "Credible Oil Supply Contracts". He says: "The world oil market differs from the conventional text-book commodity market in three important respects. As oil is an exhaustible resource, current supply decisions depend on future prices, but there are no futures markets, and so suppliers must predict the future.[6] Second, the market is

---

[3] Larry Karp and David M. Newbery (1993), Intertemporal Consistency Issues in Depletable Resource, in *Handbook of Natural Resources and Energy Economics, vol. III*, edited by A.V. Kneese and J.L. Sweeney, *Elsevier Science Publiushers B.V.*

[4] David Newbery (1981), Oil Prices, Cartels, and the Problem of Dynamic Inconsistency", *Economic Journal*, 91 (363), pp. 617 – 646.

[5] A closed-loop equilibrium is a Nash equilibrium with strategies that depend both on time and the state.

[6] Comment by JV: The time period between deciding to start operating an oil field (and building up production capacity) and when to start extracting may be very long, up to 10 years.

dominated by a small number of large suppliers who face competition from a fringe of small producers. The market structure is neither one of pure monopoly, nor perfect competition, but closer to the more complex form of a dominant producer (in this case a cartel) facing a competitive fringe. Finally, oil is a highly politicised commodity, conferring substantial scarcity rents to those in a position to exploit them, and traded between sovereign nation states. In such a situation, the only credible long term supply contracts are those which continue to be mutually advantageous, for there is no supranational legislature to ensure that such contracts are honoured."

The issue of dynamic inconsistency can the illustrated by having a cartel, facing competition by a fringe, when the cartel wants to produce after the fringe has exhausted its reserves, and from then on operate as a monopolist. In that case we can imagine that the cartel will announce that it will sell oil at a low price in the future, which might induce the fringe to sell oil early and therefore speed up extraction so that the fringe reserves are depleted sooner. After the fringe is left with no reserves and hence left the market, the cartel remains the sole producer acting as a monopolist, having an obvious incentive to renege on the former incredible promise of selling at a low price, and rather raise the price. In a rational equilibrium, the fringe will see through this line of reasoning and will not find the original cartel announcement credible, and hence will not speed up extraction early. The outcome sketched above cannot be part of equilibrium. (If a price path should make a jump at some point in time in the future – a jump foreseen by the players having rational expectations – such a price path cannot be part of equilibrium, because some producers will have an incentive to change their initial plans. As long as there are incentives to revise the plans announced earlier, we cannot have an equilibrium. This is similar to the problem we discussed in Lecture #3 (Sept 7), with sequential extraction from several deposits having different extraction costs, where we explained why the equilibrium price path is continuous.)

a) *A dominant firm – competitive fringe model: The static case*

Consider the following market structure: We have a profit-maximizing cartel and a competitive fringe (consisting of a number of small, independent producers, each maximizing profit as price taker). The cartel is a leader, in the sense of setting the price that is taken as exogenous by the fringe, so as to maximize the profit given the fringe's supply response. The fringe members take their output decisions after the dominant firm has announced a price.

Suppose market demand is known by all parties to be $D(p)$, with $D'(p) < 0$, and a supply *response* taken by the fringe as given by $S(p)$, being increasing in the price set by the cartel. This supply function is common knowledge. The cartel's objective is to maximize the profit, as a monopolist on the *residual* demand curve, given by the difference between market demand and the supply of the fringe; $D(p) - S(p)$, when assuming no cost of production for the cartel. (This residual demand facing the cartel can be zero if the price is set so high that the fringe takes over the entire market, or can be equal to the market demand if the price is below the lowest price compatible with any production by the fringe; i.e. below a lower bound $\underline{p}$ for which $S(\underline{p}) = 0$.) The monopolist therefore will choose a price so that $p[D(p) - S(p)]$ is maximized. If we suppose that $p[D(p) - S(p)] := v(p)$ is concave in p, the profit-maximizing price, $\hat{p}$, will obey $v'(\hat{p}) = D(\hat{p}) - S(\hat{p}) + p\left[D'(\hat{p}) - S'(\hat{p})\right] = 0$; i.e., marginal revenue derived from the *residual* demand curve set equal to zero. If this price is compatible with positive production by the fringe, the market is divided between the two groups of producers. This is the Stackelberg-outcome. (This outcome is easy to illustrate.)

Within this setting, it was assumed that the dominant firm will announce the price $\hat{p}$ and the fringe then responds by producing $S(\hat{p})$, known, of course, with perfect certainty by the cartel. When the model is to be applied for the world market for oil, it may not be very useful. This point has been elaborated in a paper by Michael Hoel; see Hoel (1983) in a variant of the model above, but with a different decision sequence:[7] Rather than assuming the sequence above, with the subsequent response behavior by the fringe, he assumes that the fringe (for some reason, say the time it takes to build up production capacity) has to decide on the level of output before the dominant firm sets the price.[8] Instead of taking the stipulated or announced price as given and then decide on output, the fringe now has to guess on a price, call this price conjecture for p^*, and makes a supply decision, according to $S(p^*)$. Given this *predetermined* output level, the cartel will choose a price, so as to maximize $w(p;p^*) := p\left[D(p) - S(p^*)\right]$, with $S(p^*)$ now being given. (The predetermined output of the fringe works like a uniform negative

[7] Michael Hoel (1983), Monopoly Resource Extractions under the Presence of Predetermined Substitute Production, *Journal of Economic Theory*, 30 (1), pp. 201 – 212.

[8] This might be realistic when it comes to whether we should start drilling outside Lofoten, or as remarked in an unpublished note by Michael Hoel, when the producers of shale oil or oil sand decide to invest.

shift in demand facing the dominant firm.) Let the solution to this problem be given by the price p^{**} obeying $w_p(p^{**};p^*) = D(p^{**}) - S(p^*) + p^{**}D'(p^{**}) = 0$. This price choice will depend on the fringe's guess, and with $p^{**} \neq \hat{p}$. We also have $v(\hat{p}) > w(p^{**};p^*)$; see below. In the leader-follower case the dominant firm will have a wider opportunity set than if the fringe precommits to a given output.

Now, we assume that all players are fully rational in the sense that any player can "see through the entire game", and each player can put herself in the other players' shoes and calculate what one's best strategy against all the others' strategies will be. Then what each player expects the other players will choose, will in fact be realized or confirmed in equilibrium. (If we have reached equilibrium, no player will regret.) The equilibrium can therefore be regarded as a Nash-Cournot equilibrium.

Let the fringe have rational expectations (here in the sense of perfect foresight because all features of the game are known by all players), with $p^* = p^{**}$. In that case the optimal price set by the cartel, given the *predetermined* output level by the fringe, or given the fringe's strategy, will obey the first-order condition $w_p(p^{**};p^{**}) = 0$, with $p^{**} > \hat{p}$. To see this, note that we can consider

$$v'(p^{**}) = D(p^{**}) - S(p^{**}) + p^{**}(D'(p^{**}) - S'(p^{**})) = w_p(p^{**};p^{**}) - p^{**}S'(p^{**}) = -p^{**}S'(p^{**}) < 0$$

because $w_p(p^{**};p^{**}) = 0$. Hence, as $v(p)$ is concave, by assumption, we have $p^{**} > \hat{p}$. The dominant firm will set a higher price when the fringe commits to a given output than what is the case when the fringe responds to an announced price. (Such commitment will in this model relax the competitive pressure.)

To see what dynamic inconsistency means we can now use a very elegant line of reasoning provided by Hoel (1983). Suppose that the dominant firm makes a preliminary price announcement before the fringe makes her output decision. Let this announcement be $\hat{p}$. If the fringe (naïvely) believes that this will be the final price, her output will be $S(\hat{p})$. Also, if the dominant firm should stick to this price, his profit will be $v(\hat{p})$. Hence it seems that by announcing the price $\hat{p}$, the dominant firm will be able to avoid the less profitable outcome $w(p^{**};p^{**})$. But now we come to the clue: Despite this option, *the dominant firm can make it even better!* The argument above was based on *naïve beliefs* by the fringe and also irrational behavior by the dominant firm. Given the output set by the fringe, $S(\hat{p})$, we have seen above that the price being the solution to the problem, $\arg\max_p \; p\left[D(p)-S(\hat{p})\right]$, will be different from $\hat{p}$![9] The preliminary announcement $\hat{p}$ is therefore not dynamically consistent. The reason is that *if* the fringe should believe in this price announcement, the dominant firm will, according to what we have demonstrated above, benefit from deviate from that announcement. And because we have no higher legal authority enforcing binding contracts, a fringe consisting of rational players will correctly anticipate or predict that that the dominant firm will deviate from any price announcement different from p^{**}. Whatever price announcement should be made, the fringe will correctly predict the price to be p^{**} and decide its output equal to $S(p^{**})$. The dominant firm cannot then do better than by setting $p = p^{**}$.

b) A dominant firm – competitive fringe model: The dynamic case[10]

Consider now extraction over time, and let us look for a dynamic equilibrium, when the cartel and the fringe operate simultaneously, for some time period. After the fringe has exhausted its reserve, the cartel operates as a monopoly from then on until exhaustion when the terminal price hits the price ceiling, given by the cost of supplying, competitively, a perfect substitute – a "backstop" technology. In order to see what might happen in the mixed market structure we use what we have learnt about equilibrium extraction both under perfect competition and pure monopoly, when we make use of Hotelling's "No-Arbitrage Principle" to derive conditions for a dynamic equilibrium as balancing the cost and benefit of delaying extraction so that no producer has any incentive to alter its intertemporal behaviour. Extraction costs are ignored, and it is assumed that the cartel's reserve is larger than the reserves of the fringe producers. Also, there is a perfect substitute supplied competitively at a price b (a backstop technology which puts an upper limit on the price of the resource). Demand is time-invariant, and the demand elasticity is higher in absolute value the higher is the price (or lower is output sold in the market). All market participants have the same discount rate *r*; say the rate of interest in a perfect international capital market.

Both the dominant firm and each fringe producer maximize their present discounted value of future profits. The dominant firm, taking the fringe's supply response as given,

---

[9] The "argmax" means the value of the argument p that maximizes the function.

[10] An important reference is Stephen W. Salant, 1976, Exhaustible Resources and Industrial Structure: A Nash-Cournot Approach to the World Oil Market, *Journal of Political Economy*, 84 (5), pp. 1079-1094.

is maximizing with respect to the residual demand. The price path, $p(t)$, set by the dominant firm initially when taking into account the behavior of the fringe producers who take the price path as given, while the price clears the market at any point in time. What are the characteristic features of the (open-loop) Nash equilibrium price path in this mixed market context?

First, as long as there is some extraction the price cannot exceed b (a higher price will give zero demand). Secondly, because the market participants are rational, the equilibrium price path must be continuous, showing no jumps, as we have explained above. Third, with potential speculators operating, the rate of increase in the price (equal to the rent) cannot exceed r. (If not, speculators would buy up as much as possible to capture a capital gain.)
As long as the fringe has some reserves left, the equilibrium price path must obey the Hotelling Rule $\frac{\dot{p}(t)}{p(t)} = r$. If it increases at a rate below r at some point in time t, the fringe would extract and sell the entire stock so as to make a higher profit by purchasing assets, with an infinite resource supply – with excess supply, forcing the price down to zero.
During some initial period, with price below b, both the fringe and the cartel will supply the market, with the price increasing at a rate equal to r, but also the cartel's marginal revenue, derived from the residual demand curve, increasing at the same rate. As long as the fringe is adjusting its supply as the price is increasing with a smaller and smaller market demand, it is possible to have both marginal revenue and the price increasing at the same rate as long as the fringe has some reserves left.[11] At some point in time the fringe will have exhausted its stock. From then on, the cartel will be the sole supplier and will act as a monopolist for the period remaining until price being equal to b, with equilibrium (no-arbitrage) characterized by the intertemporal monopoly equilibrium condition $\frac{\dot{m}}{m} = \frac{\dot{p}}{p} + \frac{\dot{\gamma}}{\gamma} = r \Rightarrow \frac{\dot{p}}{p} < r$, where marginal revenue is $m = \gamma p$, with $\gamma := 1 - \frac{1}{\varepsilon}$, and where ε is the absolute value of the demand elasticity, with $\varepsilon > 1$ by assumption. It is assumed that $\gamma(t)$ is increasing as price is increasing over time.

[11] See Salant (op.cit.) for details.

Hence, the dynamic equilibrium is made up of two phases: The first one, with both the cartel and the fringe supplying the market, where both the equilibrium price and the cartel's marginal revenue (derived from the residual demand) will increase at a rate equal to the rate of interest. From the point in time when the fringe's reserves are depleted, we have the intertemporal monopoly equilibrium, as analyzed in Stiglitz (op.cit.).

Compared to the optimal solution (competitive equilibrium under complete set of forward markets), the resources are exhausted once price equals *b*, with the initial price set so that $p(t) = p^C(0)e^{rt}$, $p(T^C) = b$ and $\int_0^{T^C} x^C(t)dt = S$, where market demand is derived from $U'(x) = p$ and S the total initial reserves.
Under pure monopoly, we know that price is increasing at a lower rate; hence for the same amount S of resources to be extracted, the initial price $p^M(0) > p^C(0)$, and with $p^M(T^M) = b$ for $T^M > T^C$.
In the dynamic dominant firm – competitive fringe equilibrium, there is competition in between the two extremes above, with an initial price $p^{CF}(0) \in \ p^C(0), p^M(0)$, as the fringe exhausts its stock earlier than the dominant firm, and we have the monopoly equilibrium path in the last phase. The corresponding monopoly price will hit the backstop price at $T^{CF} \in \ T^C, T^M$.

An interesting feature is that if a group of price takers should form a cartel and then act like a dominant firm, with some outsiders –acting like the fringe above – then according to the findings above, the resulting equilibrium will not only benefit the cartel members, but also benefit the (free-riding) fringe producers.

c) Oligopolistic extraction over time

We will just sketch the main principles for a dynamic duopolistic equilibrium for a resource market. Each resource owner knows that how much I want to sell will affect the payoff or profit accruing to the other player, through the market price as given by $p(X) = p(x_1 + x_2)$, declining in X, where x_i is extraction of player no. i, with $i = 1,2$. The game is a non-cooperative game, with rational players and a rational expectations Nash equilibrium must rely on being self-enforcing.

Suppose that resource owner #i has a known resource deposit initially as given by S_{0i}, and ignore extraction costs. Both producers have the same discount rate r, and an infinite planning horizon. All relevant features of the model are known to both producers. We then know that the resource constraint can be expressed as

$S_i(t) = S_{i0} - \int_0^t x_i(s)ds$, as the remaining reserve of resource stock i at some point in time t.

As seen from ex ante the resource owner i's objective function; PDV of future profits, is given by $\int_0^\infty e^{-rt}\left[p(x_i(t) + x_j(t)) \cdot x_i(t)\right]dt$ which is to be maximized subject to

$\dot{S}_i(t) = -x_i(t)\; S_i(0) = S_{i0}$ and $\lim_{t\to\infty} S_i(t) \geq 0$.

Each player will now be able to influence the price, but no producer has full control of it as in the monopoly case. Here we assume that each producer is able to solve the entire game ex ante (no cooperation), as if each resource owner is able to see through the entire game during the entire horizon. This is equivalent to saying that at the ex ante stage, each resource owner announces her entire plan on $\left[0,\infty\right.$. What I do is the best response to what other player is doing, in the same way that what the other is doing is the best response to what I do. Mutual best responses; my expectations are confirmed ex post.

Technically this means that each player can solve for all the conditions characterizing the dynamic (open-loop) Nash equilibrium.

The CV Hamiltonian for player i is then:

$$H_i(x_i, S_i, x_j, \lambda_i, t) = p(x_i + x_j)x_i - \lambda_i x_i$$

Any rational player can now solve all the conditions that must hold in a dynamic equilibrium, with $X^0 = x_1^0 + x_2^0$ and price $p(X^0)$:

$\frac{\partial H_i}{\partial x_i} = p(X^O) + x_i^O p'(X^O) - \lambda_i \leq 0 \;\; (= 0 \; if \; x_i^0 > 0)$ or $m_i(x_i^O(t), x_j^0) \leq \lambda_i$ for $i, j = 1,2;\; i \neq j$.

where m_i is resource owner i's marginal revenue, and λ_i the current shadow value of her resource. Suppose that both producers are active so that $m_i(x_i^O(t), x_j^0) = \lambda_i > 0$. Each shadow price or costate variable must obey

$$\dot{\lambda}_i(t) = r\lambda_i(t) - \frac{\partial H_i}{\partial S_i} = r\lambda_i(t) \Rightarrow \lambda_i(t) = \lambda_i(0)e^{rt}$$

(Marginal cost of delaying is equal to the marginal benefit from delaying extraction.)

Hence we get: For any point in time, the equilibrium output sold at the market at some point in time t, must obey:

$$m_1(x_1^0(t), x_2^0(t)) = \lambda_1(0)e^{rt}$$

$$m_2(x_1^0(t), x_2^0(t)) = \lambda_2(0)e^{rt}$$

where the initial shadow values are determined so that the resource constraint is binding as time goes to infinity. For each resource owner marginal revenue is equal to the current shadow value of the remaining resource deposit, and we have that

$$\frac{\dot{m}_i}{m_i} = r \text{ and } m_i = p(X^0)\left[1 + \frac{x_i^0}{X^0}\frac{X^0}{p(X^0)}p'(X^0)\right] := p(X^0)\left[1 - \frac{\alpha_i}{\varepsilon(X^0)}\right] := \gamma_i p(X^0).$$

Here we have, as before, ε as the absolute value of the market demand elasticity, and $\alpha_i := \frac{x_i^0}{X^0}$ is producer i 's market share, and with the ratio between marginal revenue and price, γ_i, depending on the player herself. Note that we have $\frac{\dot{m}_i}{m_i} = \frac{\dot{p}}{p} + \frac{\dot{\gamma}_i}{\gamma_i} = r$. If there are cases where $\dot{\gamma}_i(t) = 0$, as in the case with identical resource owners $S_{10} = S_{20}$ and iso-elastic demand, then the dynamic duopoly equilibrium will coincide with the dynamic competitive equilibrium.

Consider the case where resource owner 1 has a larger initial reserve than resource owner 2; $S_{10} > S_{20}$. We also have a choke price, *b*, above which all demand will vanish. Then we know that $\lambda_1(0) < \lambda_2(0)$, and from the two equilibrium conditions, with inequality if no extraction, we have:

$$p(X) + x_1 p'(X) = m_1(x_1(\mathrm{t}), x_2(\mathrm{t})) \le \lambda_1(0)e^{rt}$$
$$p(X) + x_2 p'(X) = m_2(x_1(\mathrm{t}), x_2(\mathrm{t})) \le \lambda_2(0)e^{rt}$$

If both resource owners are active we must have – on taking the difference between the two above:

(*) $\quad -p'(X)\left[x_2 - x_1\right] = e^{rt}\left[\lambda_1(0) - \lambda_2(0)\right]$

Because $\lambda_1(0) < \lambda_2(0)$, due to $S_1^0 > S_2^0$ and no extraction costs, it follows from (*) that $x_2(t) < x_1(t)$, saying that the resource-rich owner extracts most as long as both are extracting.

The next question: Will they stop extracting at the same point in time? Will $x_1(T) = x_2(T) = 0$ and $S_1(T) = S_2(T) = 0$?

Suppose that $p'(0)$ is finite, and suppose $x_1(T) = x_2(T) = 0$. Then, with inequality in (*) we have: $-p'(X)\left[x_2 - x_1\right] = 0 \le e^{rT}\left[\lambda_1(0) - \lambda_2(0)\right]$, a contradiction, and we cannot have $x_1 = x_2 = 0$ for the same T. The two deposits will be depleted at different points in time; with $T_1 > T_2$, where $x_1(T_1) = 0$ with $S_1(T_1) = 0$ and $x_2(T_2) = 0$ as well as $S_2(T_2) = 0$. Why is $T_1 > T_2$?

This can be seen from the following line of reasoning: For $x_i = 0$, we have $m_i = p(X)$. As one deposit is gradually being depleted, we are moving up along the marginal revenue curve for that producer, and once the stock is exhausted, the marginal revenue has reached the price. Because $\lambda_2 > \lambda_1$, we must have $m_2 > m_1$, and resource owner 2 will reach the price path first; hence $T_2 < T_1$. Also, because of the choke price, equilibrium will then require that resource owner 1 will have depleted her reserves at T_1, when the price has reached b; i.e. $p(x_1(T_1)) = b$.