Seminar 1 – ECON 4925 Resource Economics - September 5, 2016

Problem 1

Show within a very simple two-period model how you would share the eating of a cake of known size, when preferences (as seen from the beginning of the first period) are given by $V(x_1, x_2)$, with V strictly increasing and strictly quasi-concave, and with $\sum_{j=1}^{2} x_j \leq S$. Here x_j is consumption in period j, whereas *S* is the size of the cake. (You may illustrate the problem in a bath-tub diagram.)

a) Provide an interpretation of the first-order condition for the problem:

$$Max_{(x_1,x_2)}\ V(x_1, x_2)\ s.t. \sum_{j=1}^{2} x_j \leq S\ \&\ x_j \geq 0\ for\ j = 1,2\,.$$

b) How is the optimal allocation affected by a higher resource base?

c) Suppose that $V(x_1, x_2) := u(x_1) + \beta u(x_2)$, where the *u*-function is strictly increasing and strictly concave, and $\beta \in\ 0,1$ is a one-periodic discount factor. How is the allocation affected by a higher discount factor?

Problem 2

Under certain assumptions, with total extraction cost being given as $C(x(t), t)$, with *C* being strictly increasing and convex in extraction *x*, and also depending on time *t* itself (to capture technological progress). With demand at t given by $x(t) = D(p(t), t)$, extraction will be determined from an optimality condition as given by $p(t) = C_x(D(p(t), t), t) + \lambda(t)$ where the costate variable $\lambda(t) = \lambda(0)e^{it}$, where i is the relevant discount rate. Use this information to provide some tentative conclusions as to what factors might affect the price over time. (Compare your conclusions with the simple version of the Hotelling rule.)