Seminar 3 – ECON 4925 Resource Economics – October 10, 2016

Problem 1

Consider monopolistic extraction of a non-renewable resource, where demand is given by $p(x)$, with positive demand no matter what the price is, with $p'(x) < 0$, extraction cost is $c(S)x$, where $S(t) = S_0 - \int_0^t x(t)dt$ is the remaining reserve at t, when S_0 is the initial reserve. We assume that $c'(S) \leq 0$ .The monopolistic owner's goal is to maximize the PDV of future profits, subject to the resource constraint and that $S(t) \geq 0 \; \forall t$.

a) Derive properties of the optimal extraction path for a monopolist when $c(S) = c$ (a positive constant).
b) How is extraction being affected by having a stock-dependent average cost of extraction with $c'(S) < 0$?

Suppose the current monopolist is facing a competitor, with a resource base smaller than the remaining reserve for the former monopolist. Suppose both have the same constant cost of extraction, set equal to zero for ease of exposition, and that there is a choke price *b*.

c) Try to model the new situation as a duopoly and look for a Nash-Cournot equilibrium when both players commit to an extraction path for the entire future. (Look at Harstad & Liski; pp. 302-304, "Suppliers with Market Power" under Private Resources, for some help.)

Problem 2

A resource-rich country has initially a known reserve of an exhaustible resource given by $S(0) = S_0$. This resource can be used in domestic production to produce a consumption good and/or it can be used directly for export so as to finance import of the consumption good. Let $x(t)$ be the domestic use as input, and $E(t)$ be the rate of export. Domestic production of the consumption good is characterized by a standard production function (strictly increasing and strictly concave) $F(x)$, with $F(0) = 0$ and $F'(0) = \infty$, whereas foreign demand for the natural resource is given by $p(E)$, with $p'(E) \leq 0$. The country has net financial capital as given by $W(t)$, and saving is given by the difference between disposable income, $F(x) + \theta W(t) + p(E)E$ and consumption;

hence $\dot{W}(t) = F(x(t)) + \theta W(t) + p(E(t))E(t) - c(t)$. The rate of return on foreign assets is given by θ.

Suppose that the planner of this country wants to maximize $\int_0^\infty e^{-rt}U(c(t))dt$, where U is strictly increasing, strictly concave and with $U'(0) = \infty$, and r being a positive utility discount rate; subject to $\dot{S}(t) = -(x(t) + E(t))$, with $S(t)$ as remaining reserves at t, and $\dot{W}(t) = F(x(t)) + \theta W(t) + p(E(t))E(t) - c(t)$, with initial foreign debt $W(0) = W_0 < 0$, and with terminal constraints as $\lim_{t\to\infty} S(t) \geq 0$ and when requiring $\lim_{t\to\infty} W(t) \geq 0$.

a) Derive the conditions for an optimal program, with (c, x, E) as (non-negative) control variables, and (S, W) state variables.
b) Interpret the various conditions, both for static and dynamic optimality.
c) What is the impact on the program of a lower rate of return on foreign assets?
d) Suppose that the rate of return on foreign assets is stochastic in the following sense: The current rate of return is θ as above, but it is fully recognized by all that it will jump downwards to a very low level, $\varepsilon > 0$, at some time into the future (as seen from the outset of the planning period). The point in time when this event will take place is therefore considered as a random variable with a well-defined probability distribution. How will this kind of uncertainty affect the planning solution from above? (Hint: Read carefully section IV in Dasgupta, Eastwood and Heal – despite the typos – for more details, if you want to try this problem. Unfortunately, we did not have time to present this idea in class Sept. 28.; we come back to this issue on Oct 12.)