From Common to Private ownership

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September 2018

We want to show that $\widehat{w}$ is lower than $\widetilde{w}$, namely that the wage under Commons Ownership is higher than the wage under Private Ownership.

Two Valleys called 1 and 2 with workers n_1 and n_2 produce output $F_1(n_1)$ and $F_2(n_2)$.

Where R_i is the rent in village i, n_i is the number of workers in village i and β is the share of rent that is intended for

Commons:

$$\frac{F_1(\widetilde{n}_1)}{\widetilde{n}_1} = \frac{F_2(\widetilde{n}_2)}{\widetilde{n}_2} = \widetilde{w} \quad and \quad \widetilde{n}_1 + \widetilde{n}_2 = L \tag{1}$$

In other words

$$F_1(\widetilde{n}_1) + F_2(\widetilde{n}_2) = (\widetilde{n}_1 + \widetilde{n}_2)\widetilde{w} = L\widetilde{w} \tag{2}$$

Privatization

$$R_1 = F_1(\widehat{n}_1) - \widehat{w}\widehat{n}_1, R_2 = F_2(\widehat{n}_2) - \widehat{w}\widehat{n}_2 \tag{3}$$

$$F'(\widehat{n}_1) = \widehat{w}, F'(\widehat{n}_2) = \widehat{w} \tag{4}$$

assuming that wages are equal to marginal productivity of labor

$$\beta(R_1 + R_2) = \widehat{w}\widehat{n}_s, R_1 + R_2 = R \tag{5}$$

and $\widehat{n}_s$ is the number of workers at non agricultural activities

Show that $\widetilde{w} > \widehat{w}$ for all $\beta < 1$.

First, if $\widetilde{n}_i \leq \widehat{n}_i$ for at least on i (that is if there exists one valley with less workers under commons than under privatization) then $\widetilde{w} = \frac{F_1(\widetilde{n}_1)}{\widetilde{n}_1} > F'(\widehat{n}_1) = \widehat{w}$ because of decreasing marginal productivity.

Second, the alternative is $\widetilde{n}_1 \geq \widehat{n}_1$ and $\widetilde{n}_2 \geq \widehat{n}_2$.

Consider:

Commons $F_1(\widetilde{n}_1) + F_2(\widetilde{n}_2) = \widetilde{w}L \geq F_1(\widehat{n}_1) + F_2(\widehat{n}_2)$ Private, since $\widetilde{n}_i \geq \widehat{n}_i$

Since rents plus wage earnings equal total production

$= \widehat{n}_1\widehat{w} + \widehat{n}_2\widehat{w} + R > \widehat{w}(\widehat{n}_1 + \widehat{n}_2) + \beta R$

Strict inequality since $\beta < 1$ from definition of β

$= \widehat{w}(\widehat{n}_1 + \widehat{n}_2) + \widehat{n}_s = \widehat{w}(\widehat{n}_1 + \widehat{n}_2 + \widehat{n}_s) = \widehat{w}L$

So that $\widetilde{w}L > \widehat{w}L$ which leads to $\widetilde{w} > \widehat{w}$