ECON5150: Problems for Nov. 4

Again, notice it is «problems» not «exercises» in TK.

- Do problem IV.1.3 by using the basic limit theorem. (This was solved for the previous seminar using the eigenvalue approach.)
- Problem IV.1.12 (the one I messed up at the seminar) is reassigned:
  - First show the case $n = 2$ in detail – you will then see what identities you need to establish.
  - Then show it for general n by induction.
- Problem IV.4.1 (b) and (c)
- Let X have infinite state space $\{0, 1, 2, \dots\}$, and assume that $p_{ij} = 0$ except for $j = 0$ or $j = i + 1$; denote $p_{i0} = 1 - k_i \in (0, 1)$.
  a) Express $f_{00}^{(n)}$ in terms of the $\{k_j\}$ and show that $\sum_{n=1}^{N} f_{00}^{(n)} = 1 - k_0 k_1 \cdots k_{N-1}$. Use this to give a necessary and sufficient condition for transience of 0 in terms of an infinite sum. (Hint: Take ln.)
  b) Show that for $n > i > 0$, then $f_{ii}^{(n)} = f_{00}^{(n)}$. Explain why this shows that either all states are positive recurrent, or all states are null recurrent, or all states are transient.
  c) Show that $\sum_{n=1}^{N+1} n f_{00}^{(n)} = 1 + k_0 + k_0 k_1 + \cdots + \big[k_0 k_1 \cdots k_{N-2}\big] + \big[k_0 \cdots k_{N-1}(1 - k_N)\big]$ and use this to give an example where 0 is positive recurrent.
  d) Let $k_j = (j+1)/(j+2)$. Is state 0 positive recurrent, null recurrent or transient?