University of Oslo
Department of Economics

ECON5150 Mathematics 4

12 December 2007, 14:30–17:30

(Transcript of the original problem set.)

There are 2 pages of problems to be solved.

All printed and written material may be used, as well as pocket calculators.

State reasons for all your answers.

Grades given: A (best), B, C, D, E, and F, with E as the weakest passing grade.

Problem 1

Solve the stochastic dynamic programming problem:

$$\max E\Big[\sum_{t=0}^{t=T-1} -e^{-u_t} + X_T\Big]$$

subject to

$$X_{t+1} = (X_t - u_t)V_{t+1}, \quad x_0 > 0 \text{ given}, \quad u_t \in \mathbb{R} \text{ for all } t.$$

Here $V_t \in \{0,1\}$, $\Pr[V_t = 0] = 1/2$, the V_t's being independently and identically distributed.

Hint: Find $J_T(x)$ and $J_{T-1}(x)$. For general t, try the formula $J_t(x) = a_t x + b_t$, and find difference equations for a_t and b_t.

Problem 2

For two Markov chains, with states $(0,1,\ldots,5)$ and $(0,1,2)$, respectively, there are given the following transition matrices.

$$P = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{8} & \frac{1}{8} & \frac{2}{8} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad Q = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} \\ 1 & 0 & 0 \\ 0 & \frac{1}{4} & \frac{3}{4} \end{bmatrix}$$

(Cont.)

(a) For matrix P, find which states are transient and which are recurrent.

(b) For matrix P, starting from state 3, find the probability of getting trapped in state 5.

(c) Consider matrix Q. Find a stationary distribution for Q. Is it a limiting distribution?

(d) Consider again matrix P. Find $(P^\infty)_{00}$, $(P^\infty)_{33}$, and $(P^\infty)_{35}$.
$(P^\infty = \lim_{n\to\infty} P^n.)$

Problem 3

Let

$$f(x, y, z) = (1 + x^2)e^{yz^2} + (x^2 + y^2)z.$$

Define

$$Y_1 = \{(x, y) : x^2 + 2y^2 \leq 10\} \qquad \text{and} \qquad Y_2 = \{(x, y) : x^2 + xy + 2y^2 \leq 10\}.$$

Define also

$$V_1(z) = \max_{(x,y)\in Y_1} f(x, y, z) \qquad \text{and} \qquad V_2(z) = \max_{(x,y)\in Y_2} f(x, y, z).$$

(a) Prove that $V_1(z)$ is continuous.

(b) Is $V_2(z)$ continuous?

Problem 4

Show that the set $C := \{(x, y, z) : x > 0,\ xze^y \geq 1\}$ is closed.