Advanced Microeconomics

ECON5200 - Fall 2014

Principal-Agent Model

- A principal wants to delegate a task to an agent;
- Delegation benefits: Increasing returns associated with tasks' division, or by the principal's lack of time or ability to perform the task himself;
- The agent and the principal have *different objectives*;
- If the agent has no private information, then the principal could propose a contract that perfectly controls the agent's behavior $\Rightarrow$ No incentives problems;
- When the agent has *private information*, then incentives problems arise.

Principal-Agent Model

- Why a theory of contract?;
- A principal delegates an action to a single agent through the take-it-or-leave-it offer of a contract;
- One-shot relationship: No repetition is available to achieve efficiency;
- The principal proposes the contract, no bargaining issues;
- A benevolent court of law must be available. It enforces the contract and imposes penalties if one of the contractual partners adopts a behavior that deviates from the one specified in the contract.

Principal-Agent Model

Definition

A contract is a legally binding exchange of promises or agreement between parties.

- Different types of contract exist;
- *Implicit contract*: A contract that is self-enforcing. When the two parties play a game where the unique Subgame Perfect Nash equilibrium of the game corresponds to the desired outcome;
- *Explicit contract*: Whenever the desired outcome is not Subgame Perfect we need an explicit contract. Internalizing court's punishment agents do not have interest in deviating from the agreement.

Principal-Agent Model

- Problem of delegating a task to an agent with different objectives and private information;
- Which private information?
- *Moral hazard* or *hidden action*: Endogenous uncertainty for the principal;
- *Adverse selection* or *ex-post hidden information*: Exogenous uncertainty for the principal;
- Non verifiability: The principal and the agent share ex-post the same information;
- No court of law can observe this information $\Rightarrow$ agency costs.

Principal-Agent Model

Hidden action

- An agent chooses actions that affect the value of trade or the agent's performance;
- The principal cannot control those actions and they are not observable either by the principal or by the court of law $\Rightarrow$ Actions are not contractible;
- Examples: Worker's effort in performing a task, timing devoted to a task, how safely a driver drives, green-investment by regulated firms...

Principal-Agent Model

Hidden action

- With moral hazard the expected volume of trade depends explicitly on the agent's effort;
- The realized production level is a noisy signal of the agent's action;
- The principal wants to design a contract that induces the highest effort from the agent despite the impossibility of directly conditioning the agent's reward on his action.

Principal-Agent Model

Hidden action

- To make the agent responsible for the consequences of his actions the principal lets the agent bear some risk;
- *Risk–sharing/efficiency* and *rent/efficiency* trade-off.

Principal-Agent Model

Hidden information

- An agent gets access to information that is not available neither to the principal nor to the court of law;
- Examples: A tenant observes local weather conditions, experts know the difficulty of the case, regulated firms have private information on their costs,...;
- To achieve efficiency, the contract must elicit the agent's private information;
- The principal must give up some information rent to the privately informed agent;
- *Rent-efficiency* trade-off.

Moral Hazard

- The principal delegates the agent to perform a task;
- The worker chooses the intensity of effort, $e \in \{0, E\}$, to perform the task. His effort positively affects the output $q \in \{0, Q\}$;
- The principal only cares about the output and don't observe effort;
- Since the effort is costly, the principal has to compensate the agent for incurring this cost;
- The agent's compensation has to be contingent on the outcome q that is a noisy signal of effort e.

Moral Hazard

Risk-sharing/efficiency trade-off

- $\Pr\{q = Q|E\} = p_E$ and $\Pr\{q = Q|0\} = p_0$ with $p_0 < p_E$;
- The risk-neutral principal's utility $q - w$;
- The agent's utility $u(w) - e$ with $u_w > 0$, $u_{ww} \leq 0$;
- The agent's reservation $\hat{u} \equiv u(\hat{w})$;
- $p_E Q - E \geq p_0 Q$ and $p_E Q - E \geq \hat{u}$ then $e = E$ is efficient.

Moral Hazard

Timing and risk-sharing/efficiency

i. The principal offers a contract to the agent;

ii. The agent then accepts or refuses the contract;

iii. If the agent refuses the contract he gets a reservation utility $\hat{u}$. If the contract is accepted, the agent then chooses the level of effort $e \in \{0, E\}$, which is unobservable by the principal;

iv. Finally, as a result of the agent's choice, a quantity q is produced.

Moral Hazard

Full Information and risk-sharing/efficiency

- If e is verifiable then the contract can specify the desired effort, $e = E$, and the contingent transfers, $\{\underline{w}, \overline{w}\}$ with $\underline{w}$ if $q = 0$ and $\overline{w}$ if $q = Q$;
- The principal's problem is:

$$\max_{\underline{w},\overline{w}} p_E Q - \left(p_E \overline{w} + (1 - p_E)\, \underline{w}\right)$$
$$s.t. \quad : \quad p_E u\left(\overline{w}\right) + (1 - p_E)\, u\left(\underline{w}\right) - E \geq \hat{u} \quad (IR)$$

- Since the principal is risk-neutral and the agent is risk adverse, then perfect insurance, $\underline{w} = \overline{w}$ s.t. $u\left(\overline{w}\right) = E + \hat{u}$.

Moral Hazard

Incomplete Information and risk-sharing/efficiency

- If e is not verifiable, then the principal's problem is:

$$\max_{\underline{w},\overline{w}} p_E Q - \left(p_E \overline{w} + (1 - p_E)\, \underline{w}\right)$$

s.t.:

$$p_E u(\overline{w}) + (1 - p_E)\, u(\underline{w}) - E \geq \hat{u} \quad (IR)$$

$$b \equiv u(\overline{w}) - u(\underline{w}) \geq \frac{E}{p_E - p_0} \quad (IC)$$

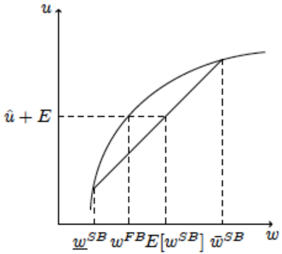
- Since $p_E > p_0$ then $\overline{w} \geq \underline{w}$ and no longer agent's full-insurance.

Moral Hazard

Incomplete Information and risk-sharing/efficiency

Using the binding constraints:

- $u\left(\underline{w}\right) = \hat{u} + E - \frac{p_E E}{p_E - p_0} < \hat{u} + E$;
- $u\left(\overline{w}\right) = \hat{u} + E + \frac{(1-p_E)E}{p_E - p_0} > \hat{u} + E$;
- $r \equiv E\left(w^{SB}\right) - w^{FB}$, risk-premium.



Moral Hazard

Full Information and rent/efficiency

- Assume that also the agent is risk-neutral, $u(w) = w$, and has limited liability, $w \geq \hat{w}$;
- The principal's problem is:

$$\max_{\underline{w},\overline{w}} p_E Q - \left(p_E \overline{w} + (1 - p_E)\, \underline{w}\right)$$

s.t.:

$$\begin{aligned} p_E \overline{w} + (1 - p_E)\, \underline{w} - E &\geq \hat{u} \quad (IR) \\ \overline{w}, \underline{w} &\geq \hat{w} \quad (LL) \end{aligned}$$

- First best solution is not affected by LL.

Moral Hazard

Incomplete Information and rent/efficiency

- Let $b \equiv \overline{w} - \underline{w}$ and $w \equiv \underline{w}$, then the principal's problem becomes:

$$\max_{b,w} p_E Q - (w + p_E b)$$

s.t.:

$$\begin{aligned} w + p_E b &\geq \hat{w} + E \quad (IR) \\ b &\geq \frac{E}{p_E - p_0} \quad (IC) \\ w &\geq \hat{w} \quad (LL) \end{aligned}$$

- IR is not an issue in the presence of LL, $\underline{w} = \hat{w}$ and $\overline{w} = \hat{w} + \frac{E}{p_E - p_0}$ and $R \equiv \frac{p_0 E}{p_E - p_0}$ is the agent's expected rent.

Moral Hazard

Inference problem

- The principal's goal is to detect what the agent has done by observing related variables;
- Should the wage increase with the observed output level? The answer is, "Not necessarily".

p	$\underline{q}$	$\hat{q}$	$\bar{q}$
$e=0$	$9/10$	$1/10$	0
$e=E$	$1/10$	0	$9/10$

Moral Hazard

Full Inference and full information (Mirrlees, 1975)

- The output is $q(e) = e + \varepsilon$, with $\varepsilon \sim F(\cdot)$ over $\mathbb{R}$, $\lim_{\varepsilon \to -\infty} \frac{F(\varepsilon)}{f(\varepsilon)} = 0$;
- P's max problem with full information:

$$\max_{e, w(q)} E\left[q - w(q) \mid e\right]$$
$$s.t. \quad : \quad E\left[u(w(q)) - e \mid e\right] \geq \hat{u}$$

- It is optimal for the P to full insure the A and $e^{FB} : h_e\left(e^{FB}\right) = 1$ with $w(q) = h(e) \equiv u^{-1}(\hat{u} + e)$.

Moral Hazard

Full Inference and incomplete information (Mirrlees, 1975)

- Consider the second-best setting and the following schedule (in terms of promised utility):

$$u = \begin{cases} U & \text{if} \quad q \geq Q \\ U - P & \text{if} \quad q < Q \end{cases}$$

- The contract is defined by $\{U, P, Q\}$;
- $q = e + \varepsilon \Rightarrow q < Q$ if $\varepsilon < Q - e$, i.e. with probability $F(Q - e)$;
- The agent's expected utility is $U - F(Q - e)P - e$;
- To implement FB $P = \frac{1}{f(Q - e^{FB})}$ with $U = \hat{u} + e^{FB} + \frac{F(Q-e)}{f(Q-e)}$;
- No cost to implement FB allocation but we need no LL.

Moral Hazard

Limited Inference and incomplete information (Mirrlees, 1975)

- $q \in [0, Q]$, $e \in \{0, E\}$ and MLRP: $l(q) \equiv \frac{f_E(q) - f_0(q)}{f_E(q)}$ with $l_q(q) > 0$;
- P's max problem:

$$\max_{w(q)} \int_0^Q (q - w(q)) f_E(q) \, dq$$

$$\int_0^Q u(w(q)) f_E(q) \, dq - E \geq \hat{u} \quad (IR, \lambda)$$

$$\int_0^Q u(w(q)) f_E(q) \, dq - E \geq \int_0^Q u(w(q)) f_0(q) \, dq \quad (IC, \mu)$$

- The FOC is $(\lambda + \mu l(q)) u_w(w(q)) = 1$, which implies that $w_q(q) > 0$.

Moral Hazard

First-Order Approach

- $q \in [0, Q]$, $e \in [e_-, e_+]$ with $F(q|e)$ and MLRP: $l(q) \equiv \frac{f_e(q|e)}{f(q|e)}$ with $l_q(q) > 0$;
- P's max problem:

$$\max_{w(q),e} \int_0^Q V(q - w(q)) f(q|e)\, dq$$

$$\int_0^Q u(w(q)) f(q|e)\, dq - \psi(e) \geq \hat{u} \quad (IR, \lambda)$$

$$e = \arg\max_{\hat{e}} \int_0^Q u(w(q)) f(q|\hat{e})\, dq - \psi(\hat{e}) \quad (IC, \mu)$$

- By using FOA, if the argmax of IC is unique and SOC are satified, then we can replace IC by FOC.