Advanced Microeconomics

ECON5200 - Fall 2014

Moral Hazard

Full Inference and incomplete information (Mirrlees, 1975)

- Consider the second-best setting and the following schedule (in terms of promised utility):

$$u = \begin{cases} U & \text{if} \quad q \geq Q \\ U - P & \text{if} \quad q < Q \end{cases}$$

- The contract is defined by $\{U, P, Q\}$;
- $q = e + \varepsilon \Rightarrow q < Q$ if $\varepsilon < Q - e$, i.e. with probability $F(Q - e)$;
- The agent's expected utility is $U - F(Q - e) P - e$;
- To implement FB $P = \frac{1}{f(Q - e^{FB})}$ with $U = \hat{u} + e^{FB} + \frac{F(Q-e)}{f(Q-e)}$;
- No cost to implement FB allocation but we need no LL.

Moral Hazard

Limited Inference and incomplete information (Mirrlees, 1975)

- $q \in [0, Q]$, $e \in \{0, E\}$ and MLRP: $l(q) \equiv \frac{f_E(q) - f_0(q)}{f_E(q)}$ with $l_q(q) > 0$;
- P's max problem:

$$\max_{w(q)} \int_0^Q (q - w(q)) f_E(q) \, dq$$

$$\begin{aligned} \int_0^Q u(w(q)) f_E(q) \, dq - E &\geq \hat{u} \quad (IR, \lambda) \\ \int_0^Q u(w(q)) f_E(q) \, dq - E &\geq \int_0^Q u(w(q)) f_0(q) \, dq \quad (IC, \mu) \end{aligned}$$

- The FOC is $(\lambda + \mu l(q)) u_w(w(q)) = 1$, which implies that $w_q(q) > 0$.

Moral Hazard

First-Order Approach

- $q \in [0, Q]$, $e \in [e_-, e_+]$ with $F(q|e)$ and MLRP: $l(q) \equiv \frac{f_e(q|e)}{f(q|e)}$ with $l_q(q) > 0$;
- P's max problem:

$$\max_{w(q),e} \int_0^Q V(q - w(q)) f(q|e)\, dq$$

$$\int_0^Q u(w(q)) f(q|e)\, dq - \psi(e) \geq \hat{u} \quad (IR, \lambda)$$

$$e = \arg\max_{\hat{e}} \int_0^Q u(w(q)) f(q|\hat{e})\, dq - \psi(\hat{e}) \quad (IC, \mu)$$

- By using FOA, if the argmax of IC is unique and SOC are satified, then we can replace IC by FOC.

Adverse Selection

This type of agency problem arises in many settings:

- Interaction between the shareholders of a firm and its managers, or the firm and its workers: Private information about the productivity of the managers or the workers;
- Interaction between an investor and a firm, or a bank and its managers: private information about the projects undertaken;
- Relationship between an insurance company and its customers: private information about the risks that the customer is facing;
- Price discrimination: private information about the customers' willingness to pay.

Adverse Selection

Price discrimination: Full Information

- P (seller) produces q at cost $C(q)$ with $C_q, C_{qq} > 0$ and sells at t;
- A (buyer) gets benefit θq with $\theta \in \{\underline{\theta}, \overline{\theta}\}$ with probabilities $\{\underline{\mu}, \overline{\mu}\}$;
- Complete info P's problem:

$$\max_{t,q} t - C(q)$$
$$s.t. \quad : \quad \theta q - t \geq 0$$

- FB allocation is $C_q\left(q^{FB}(\theta)\right) = \theta$ and $t^{FB}(\theta) = \theta q^{FB}(\theta)$.

Adverse Selection

Price discrimination: Incomplete Information

- Incomplete info P's problem:

$$\max_{(\underline{t},\underline{q}),(\overline{t},\overline{q})} \underline{\mu}\left(\underline{t}-C\left(\underline{q}\right)\right)+\overline{\mu}\left(\overline{t}-C\left(\overline{q}\right)\right)$$

s.t.:

$$\begin{array}{rcll} \overline{\theta}\overline{q}-\overline{t} & \geq & 0 & \left(\overline{IR}\right) \\ \underline{\theta}\underline{q}-\underline{t} & \geq & 0 & \left(\underline{IR}\right) \\ \overline{\theta}\overline{q}-\overline{t} & \geq & \overline{\theta}\underline{q}-\underline{t} & \left(\overline{IC}\right) \\ \underline{\theta}\underline{q}-\underline{t} & \geq & \underline{\theta}\overline{q}-\overline{t} & \left(\underline{IC}\right) \end{array}$$

- Let $\overline{r}\equiv\overline{\theta}\overline{q}-\overline{t}$ and $\underline{r}\equiv\underline{\theta}\underline{q}-\underline{t}$ the buyers' rent.

Adverse Selection

Price discrimination: Incomplete Information

- The P's problem is equal to:

$$\max_{\left(\underline{t},\underline{q}\right),\left(\overline{t},\overline{q}\right)} \underline{\mu}\left(\underline{\theta}\,\underline{q} - C\left(\underline{q}\right) - \underline{r}\right) + \overline{\mu}\left(\overline{\theta}\,\overline{q} - C\left(\overline{q}\right) - \overline{r}\right)$$

s.t.:

$$\begin{aligned} \overline{r} &\geq 0 \quad \left(\overline{IR}\right) \\ \underline{r} &\geq 0 \quad \left(\underline{IR}\right) \\ \overline{r} &\geq \underline{r} + \left(\overline{\theta} - \underline{\theta}\right)\underline{q} \quad \left(\overline{IC}\right) \\ \underline{r} &\geq \overline{r} - \left(\overline{\theta} - \underline{\theta}\right)\overline{q} \quad \left(\underline{IC}\right) \end{aligned}$$

- From $\left(\overline{IC}\right)$ and $\left(\underline{IC}\right)$ $\overline{q} \geq \underline{q}$, and $\left(\overline{IR}\right)$ and $\left(\underline{IC}\right)$ are not binding.

Adverse Selection

Price discrimination: Incomplete Information

- *Rent/efficiency trade-off*: *SB* allocation in terms of q:

$$\begin{aligned} \overline{q}^{SB} &: \quad C_q\left(\overline{q}^{FB}\right) = \overline{\theta} \\ \underline{q}^{SB} &: \quad C_q\left(\underline{q}^{SB}\right) = \underline{\theta} - \frac{\overline{\mu}}{\underline{\mu}}\left(\overline{\theta} - \underline{\theta}\right) \end{aligned}$$

- In terms of transfers:

$$\begin{aligned} \overline{t}^{SB} &= \overline{\theta}\,\overline{q}^{SB} - \left(\overline{\theta} - \underline{\theta}\right)\underline{q}^{SB} \\ \underline{t}^{SB} &= \underline{\theta}\,\underline{q}^{SB} \end{aligned}$$

- $\underline{q}^{SB} < \overline{q}^{SB}$.

Adverse Selection

General Framework

- $q \in Q$ and $\theta \in \Theta \equiv \left[\underline{\theta}, \overline{\theta}\right]$ distributed according to $F(\cdot)$;
- Agent's preference $U(q, t; \theta) = V(q, \theta) - t$;
- Principal's preference $t - C(q, \theta)$;
- P's problem under full info:

$$\max_{(t,q)} t - C(q, \theta)$$

s.t.:

$$V(q, \theta) - t \geq 0$$

- FB allocation is $C_q\left(q^{FB}(\theta), \theta\right) = V_q\left(q^{FB}(\theta), \theta\right)$ and $t^{FB}(\theta) = V\left(q^{FB}(\theta), \theta\right)$.

Adverse Selection

General Framework: Implementability

- Let $r(\theta) \equiv V(q(\theta), \theta) - t(\theta)$ the agent's rent;
- Then $r(\theta) \geq 0$ and $r(\theta) = \max_{\tilde{\theta}} V\left(q\left(\tilde{\theta}\right), \theta\right) - t\left(\tilde{\theta}\right)$;
- Spence-Mirrlees condition (i.e. single-crossing property) $V_{q\theta}(q, \theta) \geq 0$ for each q, θ;

Theorem

If single-crossing property holds, then $(q(\cdot), r(\cdot))$ is incentive compatible iff $q_\theta(\theta) \geq 0$

$$r(\theta) = r(\underline{\theta}) + \int_{\underline{\theta}}^{\theta} V_\theta(q(s), s)\, ds$$

Proof.

(See notes!).

Adverse Selection

General Framework: Optimality

P's problem under incomplete info:

$$\max_{(t(\cdot),q(\cdot))} \int_{\underline{\theta}}^{\bar{\theta}} \left[t(\theta) - C(q(\theta),\theta)\right] f(\theta)\, d\theta$$

s.t.:

$$\begin{aligned} V(q(\theta),\theta) - t(\theta) &\geq 0,\ \forall\theta \\ V(q(\theta),\theta) - t(\theta) &\geq V\left(q\left(\tilde{\theta}\right),\theta\right) - t\left(\tilde{\theta}\right),\ \forall\theta,\tilde{\theta} \end{aligned}$$

Adverse Selection

General Framework: Optimality

By using the Theorem, the P's problem is:

$$\max_{(t(\cdot),q(\cdot))} \int_{\underline{\theta}}^{\bar{\theta}} \left[V\left(q\left(\theta\right),\theta\right) - C\left(q\left(\theta\right),\theta\right) - r\left(\theta\right)\right] f\left(\theta\right) d\theta$$

s.t.:

$$\begin{aligned} r\left(\theta\right) &\geq 0, \forall \theta \\ r\left(\theta\right) &= r\left(\underline{\theta}\right) + \int_{\underline{\theta}}^{\theta} V_{\theta}\left(q\left(s\right),s\right) ds, \forall \theta, \tilde{\theta} \\ q_{\theta}\left(\theta\right) &> 0 \end{aligned}$$

Additional assumption $V_{\theta}\left(q\left(\theta\right),\theta\right) \geq 0$. (See notes!).