1 Seminar II

Ex. 1 Consider a risk-neutral principal who delegates a task to a risk-neutral agent protected by limited liability. His effort e is a continuous variable which costs him $\psi(e)$ with $\psi'(e) > 0, \psi''(e) > 0$. The return to the principal q follows the distribution $F(\cdot|e)$ with density $f(\cdot|e)$, such that the monotone likelihood ratio property holds, i.e.,:

$$\frac{\partial}{\partial q}\left(\frac{f_e(q|e)}{f(q|e)}\right) > 0$$

The principal benefits from $q - t(q)$ where $t(q)$ is the transfer he makes to the agent.

1. Characterize the first best effort
2. Write the agent's incentive compatibility and participation constraints when e is non verifiable. Use the first-order approach.
3. Write the Lagrangian of the principal's problem and optimize when the transfer belong to the interval $[0, q]$
4. Show that the optimal contract involves a cut-off q^* such that $t(q) = 0$ when $q < q^*$ and $t(q) = q$ when $q > q^*$

Ex. 2 Consider a monopoly facing a continuum $[0, 1]$ of consumers. Each consumer is characterized by his utility function, $\theta \log q + x$, where x is his consumption of good 1 (choose as numeraire) and q is his consumption of good 2 produced by the monopoly. The parameter θ can take two values $\bar{\theta}$,$\underline{\theta}$ with $\bar{\theta}-\underline{\theta}= 1$ and let ν the common knowledge proportion of type $\underline{\theta}$ consumers.

Consumers have large resources in good 1, x^*, so that their behavior is always characterized by the first-order conditions of their optimization programs.

The monopoly has a variable cost function $C(q) = cq$ and must incur a fixed cost K.

1. Determine the interior Pareto optimal allocation $q^*(\theta)$ (Note that consumers preference are quasi-linear)
2. Write the incentive compatibility constraints
3. Solve the optimization program of the monopoly under asymmetric information
4. Suppose that a government use a linear tax τ on the consumption of good 2 to control the monopoly. Assume also that the government maximizes a weighted average of consumers' utility function (with a weight 1), of monopoly profits (with weight $\sigma > 1$) and of taxes (with a weight $\lambda > 0$ and $\lambda < \sigma$). Show that the optimal tax is negative.
5. Solve points 2 and 3 when θ is uniform distributed. In particular, find the associated nonlinear price.