1 Seminar I: Game Theory

Ex. 1 There are three players $\{1, 2, 3\}$. Player 1 has the power to impose a status quo, which is denoted by the event "a". If this event realizes, then all the players obtain a payoff equal to zero. Conversely, if player 1 does not maintain the status quo, then players 2 and 3 are calling to play with possible final outcomes denoted $\{b, c, d\}$. Player's 2 strategies are represented by the events $\{b\}$ and $\{c, d\}$, whereas player's 3 strategies are represented by the events $\{b, c\}$ and $\{b, d\}$. The values of the final outcomes of the game for the three players, U_i with $i \in \{1, 2, 3\}$, are reported in the following table:

	a	b	c	d
U_1	0	-1	3	1
U_2	0	2	1	3
U_3	0	0	3	3

i) Provide both the extensive and the strategic form representation of the game.
ii) Find the NE of the game.
iii) Which of these equilibria is robust to the iterated deletion of dominant strategies?
iv) Which of these equilibria is SPE?

Ex. 2 There are n bidders whose private evaluation is $\underline{v} \leq v_i \leq \overline{v}$. Each of them make a bid $b_i \geq 0$. Moreover, each bidder only observes his own evaluation, but believes that the others' evaluations are i.i.d. and distributed according to $F \sim [\underline{v}, \overline{v}]$. Suppose that the player with the highest bid wins the auction by paying the second highest bid (second-price auction).
i) Show that $b_i = v_i$ is a weakly dominant strategy.
ii) Find the BNE of a first-price auction (i.e., the player with the highest bid wins the auction by paying his own bid).

Ex. 3 There is a Government (G) that has an investment opportunity, but it lacks the know-how to run the project in an efficient way. Thus, it can allow a Firm (F) endowed with the appropriate technology to run the project. The current period output Y solely depends on the current investment period I, as follows:

$$Y = I^{\alpha}, \ \alpha \in (0, 1)$$

Thus, the profits generated by the projects are:

$$\pi = Y(I) - I$$

i) Determine the first-best investment level and per-period profits.

Both G and F are risk neutral and maximize the net present value of cash flows over an infinite horizon with discount factors β_G and β_F, respectively.

Let assume that $\beta_G < \beta_F$. Let consider an infinite repeated game with perfect information between F and G, at each time G can confiscate the proceeds of the project and F can leave the country. Thus, in each stage-game the timing is as follows: i) G can decide whether to allow the firm investment or not, ii) if G allows the firm investment, then F chooses the investment level or whether to reject the offer, iii) if F accepts the offer and implements the project's investment, then G chooses the amount of profits' expropriation, $\tau \in [0, Y]$. If the project is undertaken by G the final payoffs are π^{aut} for G and 0 for F.

ii) Provide the extensive form representation of the game.

iii) Determine the Nash equilibrium of the static game. Is the Nash equilibrium also equal to the minmax of the stage-game? What does it imply for the characterization of the SPE of the infinite repeated game?

iv) Show that the autarky allocation is a SPE and it is Pareto dominated by any other SPE.

Let consider a stationary agreement characterized by $\tau = Y - I$, which implies that at each time G expropriates the F's profits. F has incentives to invest iff G has not incentives to deviate from the agreement by expropriating the entire output, i.e., $\tau = Y$.

v) What is the condition under which G has not incentive to deviate? (i.e., write the incentive compatibility constraint of G and determine the threshold level $\tilde{\lambda}$ such that if $\lambda > \tilde{\lambda}$, then G has incentive to expropriate the entire output and autarky is the unique SPE).

Let introduce the following notation: υ denotes the net present value, which the firm promises to deliver to the government at the beginning of period t; ω is the G discounted continuation value; $V_F(\upsilon)$ denotes the net present value of F given the promised value υ, such that:

$$\begin{aligned} \upsilon &= \tau + \beta_G \omega \\ V_F(\upsilon) &= Y - I - \tau + \beta_F V_F(\omega) \end{aligned}$$

The dynamic contracting problem can be written in its recursive formulation as follows:

$$
\begin{aligned}
V_F(\upsilon) &= \max_{I,\tau,\omega} Y - I - \tau + \beta_F V_F(\omega) \\
s.t &: \\
\tau + \beta_G \omega &\geq \upsilon & (\eta_{PK}) \\
\tau + \beta_G \omega &\geq Y(I) + \frac{\beta_G}{1-\beta_G}\pi^{aut} & (\eta_{IC_G}) \\
V_F(\omega) &\geq 0 & (\beta_F \eta_{IR_F}) \\
\omega &\geq \frac{\pi^{aut}}{1-\beta_G} & (\beta_G \eta_{IR_G}) \\
\tau &\geq \tau_{\min} = 0 & (\mu_{\min}) \\
\tau &\leq \tau_{\max} & (\mu_{\max})
\end{aligned}
$$

under the initial condition:

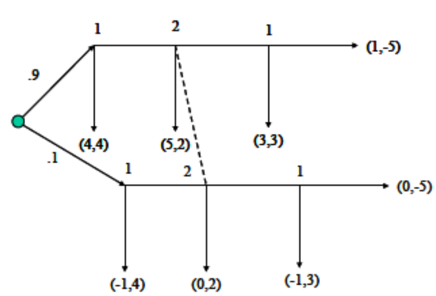
$$
\begin{aligned}
V_F(\upsilon_0) &= Y - I - \tau + \beta_F V_F(\omega) \geq 0 & (\eta_{IR_{F0}}) \\
\upsilon_0 &\geq \frac{\pi^{aut}}{1-\beta_G} & (\eta_{IR_{G0}})
\end{aligned}
$$

where the variables in the brackets represent the Lagrangian multipliers associated with each constraints.

vi) Describe the maximization program. In particular, discuss the meaning of the constraints. What does the initial condition $V_F(\upsilon_0) \geq 0$ imply in terms of current period tax payment τ?

vii) Let consider the case with $\eta_{IR_{F0}} = 0$ and $\eta_{IR_G} = 0$. Write down the Lagrangian and characterize the first order conditions with respect to the controls. Discuss how possibly binding constraints affect the equilibrium investment decision.

Ex. 4 Find the PBNE of the following game:



Note that Nature moves first!