UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Take-home exam: **ECON5200/9200 – Advanced Microeconomics**

Handed out: Wednesday, December 9, 2009 at 12:00

To be delivered by: Monday, December 14, 2009 at 2:00 p.m.

Place of delivery: Department office, 12th floor

Further instructions:

- The questions are in English, but you can give your answers in English, Norwegian, Swedish or Danish.
- The take-home exam will be marked, and the scale for the overall mark will be Pass/Fail.
- After completion, you must hand in two (2) copies of your written work. Please remember that these must not bear your name, but the individual examination number, which you will find in your studentweb.
- In addition, you must fill in the enclosed declaration
- It is of importance that the home-exam is delivered by the deadline (see above). Papers delivered after the deadline, **will not be corrected**.*)
- All papers must be delivered to the place given above. You must not deliver your paper to the course teacher or send it by e-mail.

*) If a student believes that she or he has a good cause not to meet the deadline (e.g. illness) she or he should contact the administration at the Department of Economics. Normally extension will only be granted when there is a good reason backed by supporting evidence (e.g. medical certificate).

Please do not forget the periodic course evaluation for ECON5200/9200, which you will find on the website for the course. The deadline is December 23!

ECON5200 ADVANCED MICROECONOMICS, fall 2009

General information: At least 4 of the following 6 problems has to be solved. You may discuss the problems with other candidates, but each candidate has to write a separate and independent solution.

Problem 1

Consider I consumers with Walrasian demand $x_i(p, w_i)$ where p are prices and w_i is individual wealth.

(a) What assumptions are required to write aggregate consumption as a function of prices and aggregate wealth:

$$x\left(p, \sum_{i=1}^{I} w_i\right) = \sum_{i=1}^{I} x_i(p, w_i)$$

In the following, disregard the case of given wealth distribution rules.

(b) Suppose there are two types of consumers, either $i \in A$ or $i \in B$. (The sets A and B are disjoint but their union contains all consumers). Under what condition can we write aggregate demand as a function of prices and aggregate wealth for the two types of consumers

$$x\left(p, \sum_{i\in A} w_i, \sum_{i\in B} w_i\right) = \sum_{i=1}^{I} x_i(p, w_i)?$$

Jorgenson (1990) ("Aggregate consumer behavior and the measurement of social welfare," *Econometrica* **58**, 1007-1040) derive aggregate demand under a restriction on individual indirect utility functions.

(c) How does Jorgenson's specification (3.1) relate to the discussion above?

Jorgenson also derives a welfare interpretation of aggregate demand.

(d) Are Jorgenson's assumptions fundamentally different from those invoked in proposition 4.D.1 in the textbook. Do you think Jorgenson's assumptions are strong or weak?

Problem 2

As in Blume, Brandenburger and Dekel (1991), ("Lexicographic Probabilities and Choice under Uncertainty," *Econometrica* **59**, 61-79) let Ω denote the set of states, C the set of (pure consequences) and $\mathcal{P}$ the set of objective lotteries over C, while acts are mappings from Ω into $\mathcal{P}$. A constant act is an act such that $x(\omega) = x(\omega')$ for all $\omega, \omega' \in \Omega$.

(a) Suppose $\succeq$ satisfies Axiom 1–4. Explain why the ranking of constant act determines the expected utility function? (Take the theorem of expected utility for objective probabilities for granted.)

Now, invoke Axiom 5 as well. Suppose that there are consequences $w, b \in C$ with $u(w) = 0$ and $u(b) = 1$.

(b) Demonstrate how indifference with a constant act and the act

$$y = \begin{cases} b & \omega \in E \\ w & \omega \notin E \end{cases}$$

can be used to determine the subjective probability of an event $E \subset \Omega$

Mas-Colell et al. discuss the Ellsberg paradox (p. 207). This paradox has led to an extension of theories for choice under uncertainty, using non-additive probabilities. A seminal paper is Schmeidler (1989) ("Subjective Probability and Expected Utility Without Additivity," *Econometrica* **57**, 571-587).

A simple example of non-additive probabilities with a state space $\{s_1, s_2\}$ is to assign non-additive subjective probabilities, v

$$\begin{aligned} v(\{s_1, s_2\}) &= 1 \\ v(\{s_1\}) &= v(\{s_2\}) = 0.1 \end{aligned}$$

The 10% allocated to each state represent the tiny knowledge we have, and the "missing" 80% represent genuine lack of knowledge, but $v(\{s_1, s_2\}) = 1$ as we know that one of the states for sure is the true state.

(c) Show that with the extension to expected utility suggested by Schmeidler, expected utility for an act f with the probabilities above would be (this can be derived from the first equation of section 3):

$$Eu = 0.1u(f(s_1)) + 0.1u(f(s_2)) + 0.8\min(u(f(s_1), u(f(s_2))$$

(d) Why can this kind of preferences explain the Ellsberg paradox?

Schmeidler introduces the concept of comonotonic acts and limits the independence axiom to only apply to comonotonic acts. He then derives an expected utiity representation with non-additive preferences.

(e) Are the acts used to define probabilities in point (b) comonotonic?

Problem 3

Standard Walrasian equilibrium only considers private goods. Now suppose that there is an economy where there are N non-produced private goods, and one public good produced with private goods as input.

(a) Explain why there generally will be no Walrasian equilibrium in this economy.

(b) Show how the Pareto efficient solution can be supported as an Walrasian equilibrium by extending the commodity space (Lindahl equilibrium) and discuss the welfare implication of this result.

Walker (1981) ("A Simple Incentive Compatible Scheme for Attracting Lindahl Allocations," *Econometrica* **49**, 65-71) shows that **the** Lindahl equilibrium corresponds to a Nash equilibrium in an appropriately defined game, with three or more players.

(c) Set up the game for the case of one private good (labor) and one public good, and show that the Lindahl equilibrium is a Nash equilibrium.

(d) Discuss why a Walrasian equilibrium corresponds to a Nash equilibrium, even with only three individual. Is this result likely to extend to regular Walrasian equilibrium with only private goods?

Problem 4

Consider a market where $x(p)$ is a continuous and strictly decreasing demand function for all p with $x(p) > 0$; $x(p) = 0$ for $p \geq \bar{p}$. Assume that two firms 1 and 2 have zero production costs up to capacities $\bar{q}_1$ and $\bar{q}_2$ and simultaneously set prices p_1 and p_2.

(a) What is meant by efficient rationing?

(b) Determine the equilibrium of this game for different values of $\bar{q}_1$ and $\bar{q}_2$ under the assumption of efficient rationing. Try also to determine a mixed-strategy equilibrium for some pair of values of $\bar{q}_1$ and $\bar{q}_2$ for which there is no pure-strategy equilibrium.

Kreps and Scheinkman (1983) ("Quantity precommitment and Bertrand competition yield Cournot outcomes," *Rand Journal of Economics* **14**, 326–37) consider a two-stage game where two identical firms first choose capacities and then choose prices. They show that such competition yields Cournot outcomes.

(c) What equilibrium concept is used?

(d) Give a presentation of their analysis. Why does their analysis entail that question (b) must be solved for all values $\bar{q}_1$ and $\bar{q}_2$?

Problem 5

Rothschild and Stiglitz (1976) ("Equilibrium in competitive insurance markets: An essay on the economics of imperfect information," *Quarterly*

Journal of Economics **90**, 630–49) present a model of competitive screening where competitive insurance companies compete for individuals of two types, high risk and low risk, as described in Exercise 13.D.2 of Mas-Colell et al.

(a) Explain why subgame-perfect equilibrium can be used to analyze this game, even though the insurance companies are not informed of the individuals' types? How does this conclusion change if individuals could first signal their risk type by prior behavior?

(b) Explain why there cannot be a subgame-perfect equilibrium where both types of individuals choose the same contract.

(c) Describe the subgame-perfect equilibrium in pure strategies when it exists.

(d) Explain why a subgame-perfect equilibrium in pure strategies may not exist. Under what conditions is this the case?

(e) Does a subgame-perfect equilibrium in mixed strategies always exist? What suggestions have been made in the literature concerning the market behavior of insurance firms under conditions where a subgame-perfect equilibrium in pure strategies does not exist.

Problem 6

The hidden information model of in Section 14.C of Mas-Colell et al. can be used for monopolistic screening.

(a) Provide a detailed description of the following application of the monopolistic screening model: Baron and Myerson (1982) ("Regulating a monopolist with unknown costs," *Econometrica* **50**, 911–30).

(b) Redo their analysis in the simplified case there the regulated monopolist is of two types: high cost or low cost.

Declaration

Please fill in this form and hand it in to the Department of Economics together with your exam.

I hereby declare that my **exam**, handed in for

ECON. . . . . .

at Department of Economics, University of Oslo

1. Has not been used for exsams at other educational institutions, in Norway or abroad.
2. Contains no quotations or extracts from written, printed or electronic sources without the source being referred to.
3. All references are listed in the bibliography
4. I am aware the contravention of these rules are a form of cheating, and against the rules of the University.

Oslo, date……

Students signature: ……..