## Exam in ECON5200, Fall 2020 <br> Problem 1

Weight: 1/3 (with equal weight on each subproblem)

Please, read the paper "Redistribution through Markets" by Dworczak, Piotr, Scott Duke Kominers, and Mohammad Akbarpour, forthcoming in Econometrica.

1. The authors study a two-sided market with private information. Briefly present their model. Which assumptions do you find particularly restrictive?
2. Despite the type of economy, the first welfare theorem holds: the competitive equilibrium is Pareto efficient. Explain the economic intuition of this result.
3. Explain the difference between "simple" mechanisms and "general" mechanisms. Discuss the difference between the optimal "general" mechanisms identified in Theorem 1 and the "simple" mechanisms.
4. Discuss how optimal intervention through price controls and rationing depend on same-side vs opposite-side inequality.
5. The textbook diagram below is often introduced to explain that equalizing demand and supply at some equilibrium price is welfare maximizing (maximizes the surplus of buyers and sellers). In light of your reading of Dworczak, Kominers, and Akbarpour, what is wrong with this reasoning? Illustrate graphically the optimal mechanism.

6. Propose an example in which the results in the paper may give insights for optimal policy intervention (beyond the examples in the paper). Discuss to which extent your example is likely to satisfy the assumptions of the results in the paper (as done by the authors for the examples they propose, i.e., kidney, housing, and labor).

# Exam in ECON5200/9200B, Fall 2020 

## Problem 2

## Weight: $1 / 3$ (with equal weight on each subproblem)

Please try to understand the motivation for and the approach promoted in "Perfect conditional $\varepsilon$-equilibria of multi-stage games with infinite sets of signal and actions" by Roger B. Myerson and Philip J. Reny, in Econometrica 88, 495-531 (2020).
(a) Define the concept of a sequential equilibrium. Why is this definition restricted to finite games? In a finite game of perfect information, why is a strategy profile part of sequential equilibrium if and only if it is a subgame-perfect equilibrium? What do Myerson and Reny mean by the term standard finite games?
(b) The concept of a perfect Bayesian equilibrium (PBE) has been suggested to be used also for, e.g., signaling games that are not finite by having infinite action sets. The concept of a PBE (or a weak PBE) has been defined in various ways, see, e.g., Sections 9.C and 13.C of Mas-Colell et al. Give one formal definition of a (weak) PBE.
(c) In a standard finite game, any (weak) PBE will also be a sequential equilibrium, but the converse does not hold. Give an example of a standard finite game with a (weak) PBE which is not sequential.
(d) Use Example 2.1 of Myerson and Reny (2020) to explain why one cannot define a concept of sequential equilibrium in an infinite game by taking limits of sequential equilibria of standard finite games that approximate it.
(e) Myerson and Reny (2020) observe in the introduction and again in Section 4 that, in a standard finite game, a strategy profile is part of a sequential equilibrium if and only if, for every $\varepsilon>0$, there is an arbitrarily close completely mixed strategy profile that is a conditional $\varepsilon$-equilibrium. What is a conditional $\varepsilon$-equilibrium? Use Example 5.3 of Myerson and Reny (2020) to explain why the concept of a completely mixed conditional $\varepsilon$-equilibrium cannot be the basis for a generalization of the concept of a sequential equilibrium to infinite games.

## Problem 3

Weight: $1 / 3$ (with equal weight on each subproblem)

The following questions draw on the insight developed in "A Theory of Auctions and Competitive Bidding," by Paul R. Milgrom and Robert J.. Econometrica, Sep., 1982, Vol. 50, No. 5, pp. 1089-1122.

This paper was central in the justification of The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel, 2020.

If the text is unclear, please make assumptions you find natural and make them explicit.
Suppose there is one object (e.g., a car) and two potential buyers. We start by considering "private values." In particular, buyer $i \in\{1,2\}$ has value $\theta_{i}$, which is uniformly distributed on $[0,1]$. So, $i$ 's payoff is:

$$
u_{i}=I_{i}\left(\theta_{i}-p_{i}\right),
$$

where $I_{i} \in\{0,1\}$ takes the value of 1 if and only if $i$ buys the good and if $i$, in that case, pays $p_{i}$.

1. Consider a social choice function which allocates the good to the buyer with the largest valuation. Please describe the Vickrey-Clark-Groves (VCG) mechanisms implementing such a social choice function.
2. When is VCG equivalent to a second-price auction? Explain.
3. What is the expected price the winner pays in the second-price auction?
4. Consider first-price auctions. Suppose that each buyer's strategy is to make a bid equal to $b_{i}=$ $\alpha+\beta \theta_{i}$. What are the equilibrium $\alpha$ and $\beta$ ?
5. Explain why you have now verified the paper's Theorem 0 and the "revenue equivalence" result discussed after it.
6. Consider now common values. By that, we will mean that there is a true common value of the car and that is equal $V=\left(\theta_{1}+\theta_{2}\right) / 2$. Thus, buyer $i$ only knows some of the aspects (i.e., $\theta_{i}$ ) determining the true value of the car. Consider, again, second-price auctions. What is the equilibrium bid strategy, as a function of one's own type (i.e., private information), $\theta_{i}$ ?
7. Suppose that after $\theta_{i}$ is observed by buyer $i$, the seller observes $\theta_{1}$ and $\theta_{2}$ and the seller can have a policy to reveal both valuations to everyone, before the auctions, or to never reveal any such information. Which of the two alternative policies (i.e., always reveal, or never reveal) maximizes the seller's expected profit? Please explain your reasoning, and relate your conclusion to the results in the paper
8. Is the value of the seller's information disclosure different when values are private and not common, i.e., in the setting considered in question 3? Please explain your reasoning, and relate it to the equilibrium concept that we use when we say that we implement VCG.
