ECON5200/9200 - Advnced Microeconomics - autumn 2023

Problem 1 - General equilibrium part

This problem is based on Roy Radner (1968): “Competitive equilibrium under uncertainty” *Econometrica* Vol 36, pp. 31-58.

1. How does Radner’s set up differ from a standard Arrow-Debreu equilibrium?

Focus on the example in section 7.

2. Given the equilibrium prices given in (7.10), how much labour does each consumer supply in each state of the world.
3. Write down each consumers utility maximization problem and show that the solution given does in fact solve the utility maximization problem.
4. Write down the producers profit maximization problem and show that the given equilibrium is indeed an equilibrium.
5. Suppose that all agents have full information and thus know which state has realized, would (7.10) still describe an equilibrium given the preferences and technology?

Exam ECON5200/9200B: Game theory and Mechanism design part

Problem 2 (Game theory): This question is based on the paper "Persuasion by Cheap Talk" (2010) by Chakraborty and Harbaugh in the *American Economic Review*.

1. Define the game formally as an extensive-form game with incomplete information. Note that the payoffs of the decision maker are not formally defined. You have to come up with one that respects the assumptions in the paper.

2. The solution concept used in the paper is Perfect Bayesian Equilibrium (equivalent to the weak Perfect Bayesian Equilibrium we have seen in class). Would you expect different results if Bayes Nash Equilibrium was the solution concept adopted? Why?

3. Consider the application II.A. "Persuasive Recommendations". Assume that the utility from not buying ϵ is commonly known. Can there be an influential equilibrium in this case? Can the buyer be persuaded to buy?

4. Consider the application II.B. "Influencing Voters". The application introduces a game where two voters cast a ballot simultaneously and the unanimity rule is used. In class, we have seen that "pivotal reasoning" plays an important role in the analysis of voting games. However, this doesn't play a role here. What is the key assumption that makes the issue of pivotal reasoning disappear?

5. Consider the application II.C. "Advertising as Cheap Talk". Modify the payoffs of the sender so that $U(a) = \frac{a_2}{a_1}$. Assume that $\theta_i \sim U[0,1]$ for $i = 1, 2$ and $\theta_1 \perp \theta_2$. Construct an influential equilibrium. Give an economic intuition of how this equilibrium can be sustained.

Problem 3 (Mechanism design): This question is based on the paper "Efficient Mechanisms for Bilateral Trading" (1983) by Myerson and Satterthwaite in the *Journal of Economic Theory*.

1. In the paper, the expected payoffs of player 2 of type v_2 reporting $\hat{v}_2$ are $v_2\overline{p}_2(\hat{v}_2) - \overline{x}_2(\hat{v}_2)$ where $\overline{p}_2$ and $\overline{x}_2$ are the expected probability of getting the good and the expected transfer. Suppose instead that the payoffs were $h(v_2, \overline{p}_2(\hat{v}_2)) - \overline{x}_2(\hat{v}_2)$. Give a sufficient condition on h (introduced in class) that guarantees that in any Bayesian Incentive Compatible mechanism, $\overline{p}_2$ is (weakly) increasing. Prove your claim.

2. Show how to obtain expression (4) in the proof of Theorem 1 using the Envelope Theorem of Milgrom and Segal (2002).[1]

3. Consider the following extensive-form game with incomplete information. There are two players bargaining over a good owned by player 1. Let v_i be the value of player i for the good. This value is private information and is distributed according to a strictly positive pdf f_i. In the first period, each player decides whether to participate in the bargaining game. If one of them decide to not participate, the payoffs are v_1 and 0 for player 1 and 2 respectively. If both of them accept, they play according to the following protocol. An offer is a transfer from player 2 to player 1. Whenever an offer is accepted, the object goes to player 2 and player 2 has to transfer the offer.

   - In period 1, player 1 makes an offer, player 2 either accepts or rejects.
   - In period 2, a player is drawn with probability 0.5 and can make an offer. The other player either accepts or rejects.
   - In period 3, each player makes an offer simultaneously. If player 2's offer is (strictly) larger than player 1's, the good is transferred to player 2 and the transfer is the arithmetic mean of the two offers.

   If no offer has been accepted in the first two periods or player 1's offer was larger than player 2's in the third, the game ends and player 1 keeps the good. There is no discounting and the players payoffs are linear in the transfer.

   Give conditions under which it is possible and under which it is impossible to find an ex-post efficient allocation in a weak Perfect Bayesian Equilibrium of the game described above. Prove your claim. Ex-post efficiency, in this context, is defined on p.271 in Myerson and Satterthwaite (1983).

[1] "Envelope Theorems for Arbitrary Choice Sets" (2002) by Milgrom and Segal in *Econometrica*. This is the Envelope Theorem introduced in Lecture 3.

Exam ECON5200: Game theory and Mechanism design part

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