UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON5300/9300 – Advanced Macroeconomic Theory**

Date of exam: Monday, December 20, 2010 **Grads are given:** January 6, 2011

Time for exam: 09:00 a.m. – 12:00 noon

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:
- All written and printed resources, as well as calculator, is allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

1 Dynamic programming

A country has just discovered a natural resource which yields an income per period $\bar{R}$ measured in terms of traded goods. The cost of exploitation is negligible. The government wants to maximize

$$V_0 = \sum_{t=0}^{\infty} \beta^t u(C_{N,t}, C_{T,t}) \tag{1}$$

where $C_{N,t}$ and $C_{T,t}$ are consumption of non-traded and traded goods respectively. Labor supply in the economy is exogenous and equal to $\bar{L}$. Labor inputs in the production of non-traded and traded goods are respectively L_N and L_T. However, when employment is reduced in one sector there will be some temporary unemployment and more the faster the reduction is. Thus, when employment is reduced in the traded-goods sector, the resource constraint for labor becomes:

$$L_{N,t} + L_{T,t} + \phi(L_{T,t-1} - L_{T,t}) = \bar{L}$$

where ϕ is an increasing and convex function, $\phi(0) = 0$ and $\phi'(0) = 0$. (You do not need to worry about the opposite case where non-traded employment is declining).

The production functions for the two goods are:

$$Y_{T,t} = F(K_t, L_{T,t}), \qquad Y_{N,t} = L_{N,t} \tag{2}$$

where F is homogeneous of degree one. The country can borrow and lend in international markets at a constant real interest rate (in terms of traded goods) r. The country accumulates net foreign assets, A_t, according to

$$A_{t+1} = A_t(1+r) + \bar{r} + Y_{T,t} - C_{T,t} + (1-\delta)K_t - K_{t+1} \tag{3}$$

The values of K_0, A_0, $L_{T,-1}$ and $L_{N,-1}$ are given ($L_{N,-1} = \bar{L} - L_{T,-1}$). A_t has to satisfy a no-Ponzi-game condition,

$$\lim_{t\to\infty} (1+r)^{-(t-1)} A_t \geq 0$$

while K_t cannot be negative for any t. Finally, it is assumed that the interest rate happens to be equal to the subjective discount rate, or, in other words that $\beta = 1/(1+r)$.

1. Define the value function and the Bellman-equation for this problem.
2. Use the Bellman-equation to derive the first-order conditions for the case with interior solution.
3. The first-order conditions can be summarized in the three equations

$$F_1'(K_t, L_{T,t}) = r + \delta \tag{4}$$

$$u_2'(C_{N,t}, C_{T,t}) = [\beta(1+r)]u_2'(C_{N,t+1}, C_{T,t+1}) \tag{5}$$

$$\frac{u_1'(C_{N,t}, C_{T,t})}{u_2'(C_{N,t}, C_{T,t})} = \frac{F_2'(K_t, L_{T,t})}{1 + \beta\phi'(L_{T,t} - L_{T,t+1}) - \phi'(L_{T,t-1} - L_{T,t})} \tag{6}$$

Give a brief interpretation of these.

4. Write down the conditions for a stationary equilibrium.

5. Suppose a solution has been found that satisfies the constraints and the first-order conditions. How can we know that this is actually a maximum? What additional assumptions will be sufficient?

2 Risk and risk sharing

Look at an economy that exists for two periods, 0 and 1. There are two types of agents, "capitalists" (subscript K) and "workers" (subscript W). They have preferences

$$\begin{aligned} U_K &= u_K(c_{K,0}) + \beta \mathbf{E_0} u_K(c_{K,1}) \\ U_W &= u_W(c_{W,0}) + \beta \mathbf{E_0} u_W(c_{W,1}) \end{aligned} \tag{1}$$

Here $c_{K,t}$ and $c_{W,t}$ are the period t consumption levels of workers and capitalist respectively. As usual $0 < \beta < 1$.

In questions 1 - 4 below we assume quadratic utility:

$$u_K(c_K) = (c_K - \frac{a_K}{2}c_K^2), \qquad u_W(c_W) = (c_W - \frac{a_W}{2}c_W^2) \tag{2}$$

where a_K and a_W both are positive. Higher levels of these mean that the agents are more risk-averse. (Take for granted that the economy stays within the range where marginal utility is positive).

Each worker is endowed with ℓ units of labor in both periods, but no initial capital. Each capitalist is endowed with k_0 units of capital at the start of period 0, but no units of labor.

In period 0 one unit of labor produces one unit of finished goods, while one unit of capital returns a total of κ units of finished goods after depreciation has been taken account of. Returns to scale are constant and the production function is additive in the two inputs. In period 1 one unit of labor produces Z_W units of finished goods, while one unit of capital returns a total of Z_K units of finished goods (after eventual depreciation has been taken account of). Hence, the rate of return on capital is exogenous here. From the point of view of period 0, Z_W and Z_K are stochastic variables (productivity shocks).

In period 0 finished goods can be used either for consumption or carried over to period 1 to be used as capital in production. In period 1 the whole output is used for consumption. All markets are competitive.

1. Suppose the only asset that can be bought and sold is a claim to the capital stock in period 1. A representative capitalist holds $k_{K,1}$, a worker $k_{W,1}$. Write down the budget equations for a representative worker and for a representative capitalist.

2. Show by maximizing utility that the amount of capital that the capitalists decide to carry over to period 1 can be expressed as

$$k_{K,1} = \frac{\kappa k_0 + [\beta \mathbf{E}_0 Z_K - 1]/a_K}{1 + \beta \mathbf{E}_0 Z_K^2} \tag{3}$$

3. The amount of capital that the workers carry over can in the same way be expressed as (you do not need to prove this)

$$k_{W,1} = \frac{\ell[1 - \beta \mathbf{E}_0 Z_K \mathbf{E}_0 Z_W] + [\beta \mathbf{E}_0 Z_K - 1]/a_W - \ell \beta \mathbf{Cov}_0(Z_K, Z_W)}{1 + \beta \mathbf{E}_0 Z_K^2} \tag{4}$$

   Try to interpret and compare the two expressions (3) and (4). (Hint: Look first at the case when there is no uncertainty).

   (a) What are the roles of intertemporal substitution, consumption smoothing and risk aversion?

   (b) Who will hold the most capital?

   (c) What can make the workers want to be net borrowers of capital?

   (d) If capitalists happen to be less risk averse than workers, does this tend to make the capitalists hold relatively more capital?

4. Suppose $a_K = a_W$. How are the two types of risk shared between workers and capitalists when one looks at second period consumption?

5. Suppose Z_W and Z_K can take on respectively M_W and M_K different values. How many different Arrow-Debreu commodities can be defined in this economy? How many Arrow securities are needed to make markets complete? If you were allowed to introduce just one other asset in addition to capital, what would you suggest (no proof of optimality required)?