UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON5300/9300 – Advanced Macroeconomic Theory**

Date of exam: Monday, December 19, 2011 **Grades are given: January 5, 2012**

Time for exam: 2:30 p.m. – 5:30 p.m.

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:
- No resources allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

A

In this question we shall look at a pure endowment economy with two types of consumers, A and B. There are equal numbers of the two types. Their utility functions are respectively

$$U^A = \sum_{t=0}^{\infty} \beta^t c_t^A \qquad \text{and} \qquad U^B = \sum_{t=0}^{\infty} \beta^t \ln(c_t^B) \tag{1}$$

Consumers of type A have endowments

$$y_t^A = \mu > 0, \quad t = 1, 2, 3, \ldots \tag{2}$$

while type B consumers have

$$y_t^B = \begin{cases} 0 & \text{if } t = 0, 2, 4, 6, \ldots \\ \alpha > 0 & \text{if } t = 1, 3, 5, 7, \ldots \end{cases} \tag{3}$$

Assume that $\beta\alpha \leq (1+\beta)\mu$.

1. Define a competitive equilibrium with time 0 trading (Arrow-Debreu equilibrium) in this model.

2. Find the equilibrium prices and quantities. (Disregard the possibility of corner solutions). Explain why the sum of expenditures over periods 0 and 1 must equal the sum of incomes over the same two periods for each consumer.

3. Compute the time 0 wealth of the two types of consumers.

4. Why did we assume that $\beta\alpha \leq (a+\beta)\mu$? Discuss briefly what happens to the equilibrium prices when this condition is not satisfied. (No proof is expected).

5. Define a sequential equilibrium with a complete set of Arrow-securities in this economy.

6. Compute prices and quantities in the equilibrium with sequential trading. How do they relate to the equilibrium with period 0 trading?

B

We are looking at a consumer who lives for T periods. His utility function is

$$U = \sum_{t=0}^{T-1} \beta^t \ln c_t \tag{4}$$

where c_t is consumption in period t.

The consumer's wage income is subject to two types of uncertainty. He may become disabled from one period to the next. As disabled he earns no wage income, but receives a constant benefit, b. The transition probability to disability is constant and equal to π. The transition probability out of disability is always zero. As indicator of disability status you may want to use a dummy variable, D_t which is zero when able and 1 when disabled. When able the consumer works in a cyclic industry. His wage income in period t, w_t, is $\bar{w}$ in good periods, $\underline{w}$ in bad periods. By assumption $\bar{w} > \underline{w} > b$. The transition probability between good and bad periods is q in both directions. Assume that $q = 0.5 > \pi$. The disability status and the state of industry is revealed at the beginning of the period. With our assumptions the earned income of the consumer in period t is $y_t = D_t b + (1 - D_t)w_t$.

The consumer can invest in a risk-free asset with a constant interest rate r. He can borrow at the same rate. No other assets are available. Denote the (net) amount of the risk-free asset held from period t to period $t + 1$ as a_{t+1}. The consumer starts life with $a_0 = 0$ and has to make sure that $a_T \geq 0$.

1. Write down the accounting equation that shows how the consumer accumulates assets over time.

2. What is the maximum debt $-a(t)$ that the consumer can have at the end of period $t-1$ and still be able to pay back before he dies no matter what happens?

3. Formulate the consumers choice as a dynamic programming problem with value-function and Bellman equation. Explain what a state variable is and your choice of state variables.

4. Derive necessary conditions for an interior maximum. Suppose the consumer is in period $t < T-1$. Spell out in full the consumption Euler equations relating periods t and $t+1$ (making the transition probabilities explicit in the expression). What is the consumption growth rate an already disabled consumer will choose between two subsequent periods? Explain why the expected consumption growth rate will be higher for a consumer who is employed in the first period. An economic explanation, will be valued more than a purely mathematical one.

5. What are the potential reasons for saving in the model? To what extent will you expect consumption to depend on current income versus total wealth (including both the financial assets and the expected present value of future earnings)? No maths required.

6. Suppose three consumers with the same a_t enter period t able and earning the high wage $\bar{w}$.In period $t+1$ one of the three end up disabled, one with the low wage and one with the high wage. How do you expect the relation between their consumption levels to be in period $t+1$? Use intuition. *Do not waste time searching for explicit solutions of the model. You will be disappointed.*

7. Suppose someone offers able consumers to pay an insurance premium p in period t and in return receive additional benefits Δb in period $t+1$ if they become disabled then. No further payments are made. How will able consumers value this opportunity? What is the maximum price p that they are willing to pay for the new asset that the insurance policy is? Are they willing to pay more than the fair price $p = \pi\Delta b/(1+r)$?

Formulas that you may find useful or may do without

The arithmetic mean is greater than the harmonic mean:

$$\Sigma_{i=1}^{N}\alpha_i x_i \geq \frac{1}{\Sigma_{i=1}^{N}\alpha_i \frac{1}{x_i}} \quad \text{when} \quad \Sigma_{i=1}^{N}\alpha_i = 1$$

with $=$ only when all x_i are equal.

The sum of a qeometric series:

$$1 + k + k^2 + \cdots k^{n-1} = \frac{1-k^n}{1-k}, k \neq 1$$