UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON5300/9300 – Advanced Macroeconomic Theory**

Date of exam: Wednesday, December 18, 2013 **Grades are given: January 6, 2014**

Time for exam: 9.00 a.m. – 12.00 noon

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:
- Open book exam. All written and printer resources, as well as calculator, are allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

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December 18, 2013

Exam
Advanced Macroeconomic Theory
ECON 5310

Good Luck!

Asset Pricing

Consider a model in which there are three periods $(1, 2, 3)$ and a unit mass of identical agents. There are two states, G and B in period 2 and 3. The state turns out to be G with probability p (thus, state B happens with probability $1-p$). Each agent is endowed with a tree at the beginning of the first period, which gives the right to collect its dividends (fruit) $d_{t,i}$, at the beginning in periods $t \in \{2,3\}$ if state $i \in \{G, B\}$ realizes. Each agent also receives riskless labor income e_t. All households maximize the same utility function

$$U = E\{\frac{c_1^{1-\gamma}}{1-\gamma} + \beta\frac{c_2^{1-\gamma}}{1-\gamma} + \beta^2\frac{c_3^{1-\gamma}}{1-\gamma}\},$$

where $\gamma \geq 0$ and $0 < \beta \leq 1$. Markets are competitive and complete. Make sure that you explain and interpret all your result (precisely !!).

1. What is consumption $c_{t,i}$ in period t in state i.

2. State the household decision problem when all state contingent assets are available. Compute the state-contingent prices which support the equilibrium.(Normalize the price at period 1 to be unit.)

3. Compute the price of the tree in period 1, p_1, and the prices of the tree $p_{2,B}$ and $p_{2,G}$ in period 2 in both states B and G.

4. Compute the price of a one period risk-free bond in period 1, p_1^R.

5. Assume the following specification of endowments and dividends: $e_1 = 3, e_2 = 0, e_3 = 3$ and $d_{2,G} = d_{2,B} = 3, d_{3,G} = 1, d_{3,B} = 0$. Furthermore assume $p = 1/2$ and $\beta = 1$.

i) Show that there is no equity premium in period 1 (the return on a tree and a bond are the same). Explain why. Can a different combination of period three primitives (utility function, dividend, endowment) generate a positive premium.

ii) Compute the risk-free two period return in period 1. For which values of γ is the term structure flat/ upward sloping / downward sloping. Explain why. Is there a risk-premium for the two-period bond?

Real Business Cycles

Consider a Real Business Cycle Economy, which is characterized by the solution to the social planner problem:

$$\max_{c_t, l_t} E\left(\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + A\log(1 - l_t)\right]\right)$$

subject to

$$\begin{aligned} c_t + i_t &= y_t \\ k_{t+1} &= i_t + (1-\delta)k_t \\ y_t &= k_t^{\alpha} l_t^{1-\alpha} \\ c_t &\geq 0, l_t \in [0,1], k_0 \text{given} \end{aligned}$$

1. Derive the first-order conditions of the social planner problem
2. Derive the steady-state relationships
3. Calibrate the model (at quarterly frequency), that is determine the values α, A, β and δ, using the following facts:
   - The quarterly interest rate of bonds is 2%.
   - The capital income share is 50%.
   - Assume that $\frac{I}{K} = \frac{\frac{I}{Y}}{\frac{K}{Y}} = \frac{0.5}{10}$
   - Assume that steady-state employment is $\frac{1}{2}$ of total time endowment: $\bar{l} = 0.5$
4. Assume now that al agents in the economy suddenly become more patient, i.e. β increases to 0.99. All other parameters remain unchanged. Compute the new steady-state values for consumption c, output y, labor l, and capital k. Explain your results.

Incomplete markets

Consider an economy with infinitely lived households with time-additive preferences,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \, u\left(c_t\right),$$

where the instantaneous utility function u is concave and monotone increasing. Individual labor income y_t is exogenous, stochastic, and idiosyncratic, i.e. *i.i.d.* across households. For simplicity, assume that the income process $y_t \in \Theta = \{y_0, y_1, ..., y_N\}$ is also *i.i.d.* over time (this simplifies notation but plays no other role otherwise). The economy is small and open. Agents can borrow and lend in the riskless bond traded on the world market. The world interest rate on riskless bonds is given by r_w. Assume that households cannot borrow more than an exogenous borrowing constraint $-\underline{a}$. There is no market insurance against the shocks to y_t.

The recursive formulation for the problem is then

$$v(a, y; r_w) = \max_{c,a'}\{u(c) + \beta \sum_{y' \in \Theta} v(a', y'; r_w)\pi(y')\}$$

subject to

$$\begin{aligned} c + a' &= (1 + r_w)a + y \\ a' &\geq -\underline{a} \end{aligned}$$

1. Explain (in words) the following statement: "*A necessary condition for there to exist a stationary distribution of wealth is that the world interest rate must be lower than the discount rate (i.e., $\beta(1 + r_w) < 1$).*"
2. Suppose the economy has reached the stationary distribution. Let $A(r_w)$ denote the aggregate household wealth (i.e., adding up wealth for all households) as a function of the interest rate r_w. Draw $A(r_w)$ as a function of r_w and motivate the graph you just drew.
3. Suppose a banking crisis hits this economy. This has no effect on earnings. However, it entails that banks can no longer lend to households (so the borrowing constraint suddenly becomes $\underline{a} = 0$). Those households who have debt in the initial equilibrium must then pay back immediately (for simplicity, assume that this is feasible without anyone being forced to have negative consumption).
    (a) What is the new stationary (i.e., long run) aggregate allocation of wealth, assuming that r_w remains constant? Give intuition for the result and illustrate using the graph in question 2 above.
    (b) What happens to aggregate consumption in the short run? Give intuition for the results. What would the aggregate dynamics of consumption be if the banking crisis were announced some periods in advance?