UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON5300/ECON9300 – Advanced Macroeconomic Theory**

Date of exam: Thursday, December 17, 2015 **Grades are given: 7 January 2016**

Time for exam: 09.00 a.m. – 12.00 noon

The problem set covers 4 pages (incl. cover sheet)

Resources allowed:

- Open book exam. All written and printed resources, as well as calculator, allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

1 Incomplete markets

Consider an economy with infinitely lived households with CRRA preferences,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{(c_t)^{1-\gamma}}{1-\gamma}.$$

Individual labor income y_t is exogenous and independent across households. It is i.i.d. with bounded outcomes, $y_t \in \Theta = \{y_0, y_1, ..., y_N\}$.

The economy is small and open. Agents can invest in a riskless bond traded on the world market. The world interest rate on riskless bonds is given by r_w. Assume that households cannot borrow at all. There is no market insurance against the shocks to y_t.

The recursive formulation for the problem is then

$$v(a, y; r_w) = \max_{c,a'} \left\{ \frac{(c)^{1-\gamma}}{1-\gamma} + \beta \sum_{y' \in \Theta} v(a', y'; r_w)\pi(y') \right\}$$

subject to

$$\begin{aligned} c + a' &= (1 + r_w)a + y \\ a' &\geq 0 \end{aligned}$$

1. How do consumption and savings decisions in this model differ from the Permanent Income Hypothesis model?

2. Explain why $r_w = 1/\beta - 1$ is an upper bound on the interest rate in order for a stationary equilibrium to exist in this economy.

3. Suppose $r_w < 1/\beta - 1$. Explain why this economy has a unique steady state with a stationary distribution. How would you solve (numerically) for the equilibrium stationary distribution?

4. How does the average wealth in this economy change as risk aversion γ increases? Use a diagram to motivate your answer.

5. Suppose there were financial innovation in this economy, so that all individuals got access to complete markets (while r_w remained unchanged). What would happen to the distribution of wealth and consumption in the short run and in the long run?

2 Real Business Cycles

Consider a Real Business Cycle Economy, which is characterized by the solution to the social planner problem:

$$\max_{c_t, l_t} E\left(\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + 2\log(1 - l_t)\right]\right)$$

subject to

$$\begin{aligned} c_t + i_t &= y_t \\ k_{t+1} &= i_t + (1-\delta)k_t \\ y_t &= k_t^{\alpha} l_t^{1-\alpha} \\ c_t &\geq 0, l_t \in [0,1], k_0 \text{ given} \end{aligned}$$

Note that there is no technological progress and no population growth.

1. Derive the first-order conditions of the social planner problem
2. Derive the steady-state relationships
3. Calibrate the model (at quarterly frequency), that is determine the values α, β and δ, using the following targets:
    - The quarterly marginal product of capital (before depreciation) is 4%.
    - The capital income share is 40%.
    - Assume that $\frac{I}{Y} = 0.2$, where both I and Y are quarterly data.
4. What is the capital/output ratio in this economy. Can you recalibrate the economy to obtain a capital/output ratio of 1 and at the same time still match the three targets above. If yes, how? If not, why not?
5. Assume now that all agents in the economy suddenly become more patient, i.e. β increases to 0.99. All other parameters remain unchanged. Compute the new steady-state values for consumption c, output y, labor l, and capital k. Explain your results.
6. Assume now again the above calibrated values (as in 3.) for α, β and δ. Furthermore assume that now the government spends 20% of output and collects lump-sum taxes to pay for this output. Compute the new steady-state values for consumption c, output y, labor l, and capital k. Explain your results.

3 Asset Pricing

Consider a model in which there are two periods $(1, 2)$ and a unit mass of identical agents. There are two states, B(oom) and R(ecession) in period 2. The state turns out to be B with probability p (thus, state R happens with probability $1 - p$). Each agent receives labor income e_1 in period 1 and e_2^B and $e_2^R < e_2^B$ in period 2 in states B and R respectively. Note that this is aggregate uncertainty, that is all agents have the same labor income. All households maximize the same utility function

$$U = E\{\frac{c_1^{1-\gamma}}{1-\gamma} + \beta \frac{c_2^{1-\gamma}}{1-\gamma}\},$$

where $\gamma \geq 0$ and $0 < \beta \leq 1$. Markets are competitive and complete.

1. Assume that markets are competitive and complete. That is all state contingent assets are available. All assets are in zero net-supply. What is consumption $c_{t,i}$ in period t in state i (in total 3 consumption levels)?

2. State the household decision problem when all state contingent assets are available. Compute the state-contingent prices which support the equilibrium (Normalize the price at period 1 to be one). Compute also the return of a risk-less real bond.

3. Assume now that $e_1 = pe_2^B + (1-p)e_2^R$. Does the risk-free interest rate equal $1/\beta$?. Explain why or why not.

4. Still assume $e_1 = pe_2^B + (1-p)e_2^R$. Suppose now that the risk is idiosyncratic, that is a fraction p of agents have high income e_2^B and a fraction $1-p$ have income e_2^R in period 2. Note that second period labor income is still risky when making decisions at period 1. Still assume that markets are complete. Does the risk-free interest rate equal $1/\beta$?. Explain why or why not.