UNIVERSITY OF OSLO
DEPARTMENT OF ECONOMICS

Exam: **ECON5300/ECON9300 – Advanced Macroeconomic Theory**

Date of exam: Friday, December 2, 2016 **Grades are given:** December 22, 2016

Time for exam: 09.00 a.m. – 12.00 noon

The problem set covers 5 pages (incl. cover sheet)

Resources allowed:

- Open book exam. All written and printed resources, as well as calculator, allowed

The grades given: A-F, with A as the best and E as the weakest passing grade. F is fail.

1 (In)Complete Markets

Consider an economy consisting of two types of households, red and blue, which each comprises 50% of the population. Households are infinitely lived and have preferences

$$\max \sum_{t=0}^{\infty} \beta^t \ln c_t,$$

where $0 < \beta < 1$ is a discount factor and c_t is consumption in period t.

Households can be either employed or unemployed. Only one type of household is employed each period. Namely, the blue are unemployed then the red are employed and the red are unemployed then the blue are employed. This unemployment shock is i.i.d. (i.e., uncorrelated over time).

When employed the household receives an endowment of $y = 1$. When unemployed the household receives an endowment of $y = 1/2$ (reflecting the value of home production, say).

There is no capital in the economy and assets are in zero net supply. In the initial period ($t = 0$), all households have zero wealth and the blue are employed.

1. **Complete markets**: Assume first that there are complete markets where all assets are in zero net supply. Assume that trade takes place after observing income in the first period.

   (a) Formulate the household's problem and write down the equilibrium conditions.

   (b) Solve for the competitive equilibrium:

      i. What is the equilibrium rate of return on a non-continent bond (i.e., a bond that pays one unit next period, irrespective of the state?

      ii. How does consumption move over time? And what is the consumption levels for the red and the blue households?

   (c) Is the competitive equilibrium Pareto optimal? Motivate your answer

   (d) Suppose the economy starts to fluctuate between recessions and booms, where income in booms are the same as above (1 for the employed and 1/2 for the unemployed), while in a recession income falls by 50% (so $y = 0.5$ for the employed and $y = 0.25$ for the unemployed).

i. How would individual consumption move (qualitatively) over time? [note: you do not have to solve for the levels of consumption – it is sufficient to describe the qualitative dynamics]

2. **Incomplete markets**: Consider now an alternative economy where there is only one asset – the non-contingent bond. Assume further that households are allowed to borrow up to ϕ units of the bond (so the bond holding $b_t \geq -\phi$).

   (a) Write down the households' budget constraint. What are the state variables for the household?

   (b) Explain why neither autarky (i.e., individual consumption = endowment very period) nor constant consumption over time can be an equilibrium.

   (c) Consider two households who are both employed. One household enters the period as borrowing constrained (so $b_t = \phi$) while the other has positive wealth. Define the *marginal propensity to consume* (out of labor income) as c_t/y_t. Which of the two households has the highest marginal propensity to consume? Explain the answer carefully.

2 Asset Pricing

- Consider a static (one-period) endowment economy with identical agents.
- Each agent is endowed with x units of a consumption goods (no uncertainty here).
- Each agent is also endowed with one unit of a stochastic consumption endowment with payoff z, where $E(z) = 1$ and $Var(z) = \sigma^2$ (Note that z may take negative values).
- Before consumption takes place, there is a pre-market in which the deterministic endowment x and the stochastic endowment z can be traded *before* the uncertainty on z is resolved. The equilibrium price of a unit of z is p units of x. Note that no consumption occurs in this pre-market, only asset trading. All consumption occurs after the uncertainty on z is resolved.
- If an agent buys q units of z in this pre-market at price p, her final (stochastic) consumption ill be $c = (x - qp) + (1 + q)z$. Note that q can be positive or negative (or zero).
- Assume that the utility function in this economy, u, is increasing and strictly concave (and identical across agents).

1. State the household problem.
2. Derive equilibrium consumption of x and q, the number of units of z bought. Explain your findings. What role does p play on this outcome?
3. In equilibrium, which relation will hold: $p < 1$, $p = 1$, or $p > 1$? Explain your reasoning.
4. Now assume that $u(c) = \theta c - \frac{\alpha c^2}{2}$, where $\theta > 0$ is large and $\alpha > 0$. (Note that u can take arguments of any sign, including $c < 0$.) Calculate the equilibrium price p (in the pre-market) as a function of x, θ, α, and σ^2. What assumption on θ do you need such that $p > 0$?

3 Bellman Equation

Consider a consumer with equity as the only asset who must choose every period consumption in a discrete-time dynamic optimization problem. Specifically, the consumer is described by the following sequence problem:

$$V(x_0) = max_{\{c_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

- $x_{t+1} = exp(r + \sigma u_t - \sigma^2/2)(x_t - c_t)$
- u_{t+1} i.i.d.
- $u_{t+1} \sim N(0,1)$
- $c_t \in [0, x_t], x_0 > 0.$

Here x_t represents wealth (equity) at time t and c_t represents consumption at period t. The realization of u_{t+1} is not known in period t and is learned only in period $t+1$. The consumer has a discount factor $\beta = exp(-\rho)$ and can only invest in a risky asset with expected return $exp(r) = Eexp(r + \sigma u - \sigma^2/2)$. Assume log utility:

$$u(c) = ln(c).$$

1. Explain why the associated Bellman equation is given by

   $$V(x_0) = max_{y \in [0,x]} u(x - y) + \beta EV(exp(r + \sigma u_t - \sigma^2/2)y).$$

   Explain all of the terms in the Bellman equation.

2. Guess that the value function takes the special form

   $$V(x) = \psi + \phi ln(x).$$

   Assume that the value function guess is correct, derive the consumption function:

   $$c = \phi^{-1} x.$$

   Now solve for ϕ.