Exam in econ5300

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1 Real business cycle theory (40%)

Consider a stochastic neoclassical growth model with endogenous labor supply h_t where preferences of a representative household are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where β is the discount factor and c_t is consumption. Production of a standard consumption good takes place with firms renting capital and labor from households at competitive markets. The economy is closed and aggregate production is given by

$$Y_t = Z_t (K_t)^\alpha (H_t)^{1-\alpha},$$

where GDP where K_t and H_t are aggregate values for capital and labor supply, respectively. Z_t is aggregate TFP. The capital stock moves according to $K_{t+1} = (1-\delta) K_t + I_t$, where I_t is aggregate investment.

1. Assume first that hourly wage w_t grows at a constant rate $g > 0$ so that the wage rate in period t is given by $w_t = (1+g)^t w_0$.

   Question: Explain why preferences of the type $u(c,h) = \log c - v(h)$ or $u(c,h) = \left(c^\theta (1-h)^{1-\theta}\right)^{1-\gamma} / (1-\gamma)$ is a necessary condition in order for the economy to have a balanced growth path (steady state).

   ANSWER: With these preferences the Euler equation in the no-risk case can be expressed as

   $$1 = (1 + r_{t+1}) \beta \left(\frac{c_{t+1}}{c_t}\right)^{-\gamma},$$

   where the net return to capital is $1+r_{t+1} = \alpha Z_{t+1} (K_{t+1}/H_{t+1})^{\alpha-1} + 1 - \delta$. This form of the euler equation implies that the intertemporal elasticity of substitution is independnt of the level of consumption. Moreover, the intra-temporal first-order condition is either

   $$v'(h) = \frac{w}{c}$$

   or

   $$\frac{1}{1-h} \frac{1-\theta}{\theta} = \frac{w}{c}.$$

   This form of the intraremporal FOC implies a constant labor supply when consumption is grocing at the same rate as wages.

2. Assume that the preferences of a representative household is given by $u(c,h) = \log c - h^{1+\phi}/(1+\phi)$.

   Question: Using the first-order conditions for the households of this economy, explain how labor supply reacts to shocks to Z_t. What is the Frisch elasticity of labor supply?

   ANSWER: The intra-temporal FOC is

   $$h^{\phi} = \frac{w}{c}.$$

   Holding consumption constant, this implies

   $$\Delta \log h = \frac{1}{\phi} \Delta \log(w)$$

   and, hence, a Frisch elasticity of $1/\phi$.

3. Suppose that Z_t is an AR(1) process, i.e.,

   $$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t.$$

   Question: Explain why TFP shocks ε_t have a larger propagation (i.e., impact in terms of magnifying the business cycle fluctuations) in this economy if ρ is closer to zero than if ρ is larger.

   ANSWER: Focus on the preferens of the form $\log c - v(h)$ (a similar argument will hold for the Cobb-Douglas preferences) and abstract from s teady-state growth. The Euler equation in a deterministic verison of this problem can be expressed as

   $$1 = \beta\left(\alpha Z_{t+1} k_{t+1}^{\alpha-1} h_{t+1}^{1-\alpha} + 1 - \delta\right) \frac{Z_{t+1}}{Z_t} \left(\frac{k_{t+1}}{k_t}\right)^{\alpha} \left(\frac{h_{t+1}}{h_t}\right)^{-\alpha} \frac{v'(h_{t+1})}{v'(h_t)},$$

   When ρ is close to zero, a shock ε_t has a large impact on the ratio Z_{t+1}/Z_t while the effect is small if ρ is close to 1. To see this, suppose period $t-1$ is steady state ($Z_{t-1} = 1$) and that future ε shocks are zero (so $\varepsilon_{t+1} = 0$). This implies $Z_{t+1}/Z_t = (Z_t)^{\rho-1} = \exp(-(1-\rho)\varepsilon_t)$. Intuitively, the effect of a transitory shock is larger because the effect will be relatively short-lived so the income effect is small and the substitution effect dominates.

4. In the model above, households can choose freely their desired labor supply H_t. Suppose now instead that people can work either zero hours or "full time" $\bar{h}$, i.e., $h_t \in \{0, \bar{h}\}$.

   Questions:

   (a) Write down a social planner problem where the planner chooses each period how many individuals should work $h_t = \bar{h}$ and how many should work zero hours.

(b) Show that the planner problem can be reformulated as a representative household with (possibly) different preferences from the individuals in the economy.

ANSWER (to a and b): Focus on the preferens of the form $\log c - v(h)$ (a similar argument will hold for the Cobb-Douglas preferences). Consider first the static problem where the planner problem can be expressed as

$$\begin{aligned} & \max_{n} \left\{ n \left[\log c_e - v\left(\bar{h}\right) \right] + (1-n) \left[\log c_u - v(0) \right] \right\} \\ & \text{subject to} \\ wn\bar{h} &= nc_e + (1-n) c_u \end{aligned}$$

where n is employment and c_e and c_u are consumption levels for the employed and unemployed, respectively. The FOC for n implies $c_e = c_u$. The problem can then be expressed as

$$\begin{aligned} & \max \left\{ \log c - n \cdot v\left(\bar{h}\right) \right\} \\ & \text{subject to} \\ c &= wn\bar{h} \end{aligned}$$

(c) What is the aggregate elasticity of labor supply? Explain why it is larger than the elasticity at the individual level (along the intensive margin).

ANSWER: the aggregate Frisch elasticity of labor supply is infinite. It reflects the planner's willingness to move workers in and out of employment. Since the equally-weighted planner has a utilitarian objective, this elasticity is very high. The individual's Frisch elasticity on the intensive margin is irrelevant because this margin is mute (since hours must be either zero or $\bar{h}$).

(d) Show that the optimal allocation in the planner problem can be decentralized with the use of employment lotteries each period, where individuals purchase lottery tickets that determine the probability that they must work this period and the wage and unemployment benefit they get if they end up as employed and unemployed, respectively.

ANSWER: Given a wage rate of w, consider a lottery which yields a consumption c and mandatory work $\bar{h}$ with probability n and consumption c and zero work with probability $1-n$, where c and n is the solution to the planner problem given w. The cost of this lottery is zero. The household's optimal holding of this lottery (given his/her first-order condition) is one unit. QED

(e) Discuss verbally how the allocations would change if markets were incomplete (no insurance against unemployment). How would this change affect the answer to 5c above?

ANSWER: With incomplete markets, the aggregate Frisch elasticity would depend on the density of people being close to indifferent between working and not working. This density depends in turn on the cross-sectional distribuiton of wealth. If many (few) people are close to this threshold, then the aggregate elasticity will be high (low). In general, the aggregate elasticity will be lower than under complete markets.

2 Asset pricing (60%)

Consider an economy with a representative agent with preferences

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t), \tag{1}$$

The representative household owns a tree that yields one unit of fruit of the consumption good c every period. This fruit cannot be stored between periods. Assume first that the economy is closed and that the household does not have any other sources of income.

1. **Question**: define a competitive equilibrium and calculate the return on bonds, the price of the tree (after picking the fruit), and the consumption path in equilibrium.

   ANSWER: A c.e. is defined as an allocation and a price sequence such that markets for goods, one-period bonds, and claims to the tree clear and households optimize. Market clearing requires $c_t = 1$ and zero bonds, $b_t = 0$. Individual optimization requires that the Euler equation is satisfied;

   $$q_t = \beta \frac{u'(c_{t+1})}{u'(c_t)} = \beta \frac{u'(1)}{u'(1)} = \beta.$$

   This also pins down the (ex-dividend) price of the tree:

   $$p_t = \sum_{k=1}^{\infty} \beta^k \frac{u'(c_{t+k})}{u'(c_t)} = \sum_{k=1}^{\infty} \beta^k \frac{u'(1)}{u'(1)} = \frac{\beta}{1-\beta}$$

2. Suppose that in period $t = 0$ the households experience an unexpected preference shock where they temporarily become more patient: the one period ahead discount factor increases temporarily to unity and then returns to its initial value $\beta < 1$. Namely, in period $t = 0$ the preferences are

   $$U_0 = u(c_0) + \sum_{t=1}^{\infty} \beta^{t-1} u(c_t),$$

and from period $t = 1$ and onward the preference are back to normal as in equation (1).

Question: Show how prices and allocations behave over time in equilibrium.

ANSWER: In period $t = 1$ the future discount factors are the same as in question 1, so $p_1 = \beta / (1 - \beta)$. The price in period $t = 0$ must then satisfy the Euler equation

$$p_0 = \frac{u'(c_1)}{u'(c_0)} (1 + p_1) = 1 + \frac{\beta}{1 - \beta} = \frac{1}{1 - \beta}$$

3. Assume instead that the economy is a small open economy where the households can purchase bonds at the world market. Moreover assume that the real return on one-period bonds is $1 + r = 1/\beta$. The household can also trade in claims to fruits on the tree. Assume further that the period utility function is CRRA with risk aversion γ,

$$u(c) = \frac{c^{1-\gamma}}{1 - \gamma}$$

Suppose now that in period $t = 0$ the one-period interest rate (on bonds paying off in period $t = 1$) suddenly falls to zero. But after one period, the interest rate r_t returns to $1/\beta - 1$. However, different from question 2 above, the discount factor for the household on "our" island remains constant at β (thus, the reason for the change in r_t is unrelated to our household).

Question: What is the paths for optimal consumption of the household and the equilibrium price of the tree in this economy?

ANSWER: The equilibrium price of the tree is set by the world market. Therefore, it follows the same path as in question 2 above, i.e., the price jumps to $p_0 = 1/(1 - \beta)$ in period $t = 0$ and falls back to $p_0 = \beta/(1 - \beta)$ in period $t = 1$. The Euler equation in period $t = 0$ is

$$\begin{aligned} \frac{u'(c_0)}{u'(c_1)} &= \beta (1 + r_0) = \beta \\ &\Rightarrow \\ c_1 &= (\beta)^{\frac{1}{\gamma}} c_0 \end{aligned}$$

Since $\beta < 1$, this implies that consumption must fall between period $t = 0$ and $t = 1$. From $t = 1$ and onward the consumption is constant (since $(1 + r)\beta = 1$). It follows that consumption must increase in period zero ($c_0 > 1$) and that future consumption must be smaller than 1 ($c_t < 1$ for $t \geq 1$) to finance the consumption boom in period $t = 0$.

4. Assume instead that in period $t = 0$ it is announced that the world-market interest rate will remain at zero for 10 periods, and return to $1 + r_t =$

$1/\beta$ after 10 periods (we maintain the assumption that the household's discount factor remains constant, so the changes in r_t are unrelated to the household's discount factor).

Question: What is the evolution of the price of the asset, the optimal consumption, and the ownership of the tree? You may illustrate your answer by drawing qualitative graphs

ANSWER: The initial jump in the price of the tree and in consumption will be larger than in question 3. The price of the tree will jump to $p_0 = 10 + \beta/(1+\beta)$. For the next 10 periods, consumption will fall $(c_{t+1} = (\beta)^{\frac{1}{\gamma}} c_t < c_t)$ and the price of the tree will fall by 1 each period. From period $t = 10$ and onward the price will be constant at $p = \beta/(1+\beta)$ and consumption will be constant at $c_t = c < 1$.

5. Norway has an oil fund and a fiscal rule ("Handlingsregelen") which dictates how much the government extracts from the fund. Until 2017 the rule was to spend a fixed fraction 4% of the value of the fund. In 2017 the rule was modified to spend 3% of the fund. The stated motivation for this change was that the average return on the fund was expected to be lower (reflecting the fall in the world-market interest rates after the 2008-2009 financial crisis).

   Question A: Please discuss this change in fiscal rule in light of economic theory.

   ANSWER: Suppose first that Norway's discount factor remained constant even though the world-market interest rate fell (as in questions 3 and 4 above). The optimal response to a temporarily lower interest rate would then be to increase consumption from the fund. This increase is larger the larger is the intertemporal elasticity of substitution $1/\gamma$. The fall in the interest rate also implied an increase in the value of the fund. The change in the optimal take-out from the fund depends on both the increase in stock-market value and γ. Suppose instead that the change in interest rate reflected (also) a change in Norway's discount factor. In this case it would be optimal to keep consumption constant.

   Question B: One proposal for a reform of the fiscal rule has been to just take out dividends. Would this be a good alternative rule in the presence of changes in the world-market interest rate?

   ANSWER: In the example above, the dividends-only rule would imply a constant consumption path. This would be optimal if the changes in world-market interest rate always reflected Norway's discount factor. Otherwise it would generally yield too little change in consumption.