# Exam
# Advanced Macroeconomic Theory ECON 5300

# 1 Real Business Cycles

Consider a Real Business Cycle Economy, which is characterized by the solution to the social planner problem:

$$\max_{c_t,l_t} E\left(\sum_{t=0}^{\infty} \beta^t \left[\log(c_t) + 2\log(1-l_t)\right]\right)$$

subject to

$$
\begin{aligned}
c_t + i_t &= y_t \\
k_{t+1} &= i_t + (1-\delta)k_t \\
y_t &= k_t^\alpha l_t^{1-\alpha} \\
c_t &\geq 0, l_t \in [0,1], k_0 \text{ given}
\end{aligned}
$$

Note that there is no technological progress and no population growth.

1. Derive the first-order conditions of the social planner problem

2. Derive the steady-state relationships

3. Calibrate the model (at quarterly frequency), that is determine the values $\alpha, \beta$ and $\delta$, using the following targets:

   - The quarterly marginal product of capital (before depreciation) is 2.8%.

   - The capital income share is 56%.

   - Assume that $\frac{I}{Y} = 0.48$, where both $I$ and $Y$ are quarterly data.

4. What is the capital/output ratio in this economy. Can you recalibrate the economy to obtain a capital/output ratio of 20 and at the same time still match the three targets above. If yes, how? If not, why not?

# 2 Complete Markets and Asset Pricing

---

Consider an economic environment with complete markets. Assume that households indexed by $i \in I$ trade all state-contingent claims at time 0 such that a household's maximization problem is of the following form:

$$\max_{c_t^i(s^t)} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t^0(s^t) \frac{c_t^i(s^t)^{1-\gamma}}{1-\gamma},$$

subject to

$$\sum_{t=0}^{\infty} \sum_{s^t \in S^t} q_t^0(s^t) \left[c_t^i(s^t) - y_t^i(s^t)\right] = 0 \quad \forall i \in I.$$

(a) Define an Arrow-Debreu security. Show that the Arrow-Debreu price $q_t^0(s^t)$ is a function of aggregate consumption. Proceed in the following steps: (i) formulate the Langrangean ii) derive the Euler equation of an individual household $i$, taking as given prices, $q_t^0(s^t)$, and (iii) substitute individual consumption for aggregate consumption

(b) Assume that markets are complete. Are the following statements correct? (i) "If we know average wealth, we also know the wealth holdings of all other households." (ii) "The wealth distribution does not matter for the aggregate decision of the household sector."

Suppose from now that individuals also make a labor supply choice, $h_t^i(s^t)$ (hours per period), and face an individual-specific but fixed wage $w^i$. Thus, labor income each period is $y_t^i(s^t) = w^i h_t^i(s^t)$. Assume that preferences are

$$u(c,h) = \log(c) - (1-h)^{(-1)},$$

such that the objective function of the household reads

$$\max_{c_t^i(s^t), h_t^i(s^t)} \sum_{t=0}^{\infty} \sum_{s^t \in S^t} \beta^t \pi_t^0(s^t) \left[ \log\left(c_t^i(s^t)\right) - \left(1 - h_t^i(s^t)\right)^{(-2)} \right].$$

(c) Solve for individual consumption, hours and Arrow-Debreu prices (as a function of aggregate consumption)