1 Growth

Consider an economy with a set of infinitely-lived households. Each household maximizes

$$\max \sum_{t=0}^{\infty} \beta^t \frac{(c_t)^{1-\gamma}}{1-\gamma}$$

where c_t is consumption in period t. The size of the population is constant. Output is produced by a large set of firms with a production function

$$Y_t = A \cdot K_t^{\alpha}, L_t^{1-\alpha},$$

where K_t and L_t are aggregate capital and labor, respectively.

Factor and product markets are competitive. Each household supplies one unit of labor and earn a wage w_t. Firms rent capital from households at a rate $r_t + \delta$, where δ is the depreciation rate.

1. Define a competitive equilibrium and provide the equilibrium conditions.

   Answer: list the equilibrium conditions (market clearing, firms and household optimization). Equilibrium conditions comprise an Euler equation, a resource constraint, prices equal marginal productivity, and market clearing for labor and goods.

2. Explain why the competitive equilibrium can be found by solving a social planner problem.

   Answer: From the 1st welfare theorem the competitive equilibrium is Pareto efficient. From the 2nd welfare theorem any Pareto optimal allocation can be decentralized as a competitive equilibrium with transfers.

3. Illustrate graphically the dynamics of the equilibrium. Make sure you explain all graphs.

   Answer: Draw a phase diagram. Explain the lines associated with $\dot{k} = 0$ and $\dot{c} = 0$. Use this to describe the dynamics.

4. Suppose there is a one-time increase in TFP A. Explain how the economy will adjust to the new steady state.

   Answer: the resource graph (associated with $\dot{k} = 0$) shifts up. The steady-state capital stock increases, so the Euler equation line (associated with $\dot{c} = 0$) shifts to the right. Consumption will jump to the new manifold and this jump could be up or down depending on θ. Then, consumption and capital will increase in line with the manifold toward the new steady state.

5. Suppose the households' risk aversion suddenly increases. Explain whether or not this will affect the steady state. Explain how the change in γ would affect the transition to steady state.

Answer: The Euler equation yields $u'(c_t) = (1 + r_{t+1})\beta \cdot u'(c_{t+1})$. Since there is no growth in steady state (due to A being constant), the steady state will not be affected. However, the transition path will be affected. The Euler equation can be written as

$$\frac{1}{\gamma}\log\left(\frac{c_{t+1}}{c_t}\right) = \log(R_{t+1}) + \log(\beta)$$

and it follows that the speed of convergence to steady state is larger the larger is the intertemporal elasticity of substitution, $1/\gamma$. Moreover, if the initial capital stock is lower than steady state, then an increase in γ will imply a positive jump in consumption (so that the growth rate can be slower).

2 Asset pricing

Consider a Lucas tree model: the economy has one tree with output Y_t each period. Output grows at a stochastic rate ε_t;

$$\log Y_t = \log Y_{t-1} + \varepsilon_t,$$

where ε_t is i.i.d.. Assume that GDP growth is lognormally distributed, i.e.,$\log(Y_{t+1}/Y_t) = \varepsilon_{t+1} \sim N\left(-\frac{\sigma^2}{2}, \sigma^2\right)$. Households are infinitely lived and maximize

$$\max E_0 \sum_{t=0}^{\infty} \log(c_t)$$

Households are identical and own an equal share of the tree. The budget constraint of the representative household is

$$c_t + p_t \cdot a_{t+1} = (Y_t + P_t) \cdot a_t,$$

where P_t is the price of the tree and a_t is the share of the tree owned by the household

1. Define a competitive equilibrium and write down the equilibrium conditions.

   Answer: list the equilibrium conditions (market clearing and household optimization). Equilibrium conditions comprise an Euler equation and the market clearing constraints, i.e., $a_{t+1} = 1$ and $c_t = Y_t$.

2. Solve for the competitive equilibrium allocations and prices [hint: use a planning problem]

   Answer: There are no externalities or market frictions, so the welfare theorems apply. The planner allocation associated with equal weight (which is the natural choice given the representative-agent assumption) implies

that consumption is equal to the fruit, $C_t = Y_t$ and, when asset holdings are decentralized, $a_t = 1$. The price of the tree is then given by the asset pricing equation for the tree;

$$\begin{aligned} P_t &= E\left\{\beta\frac{c_t}{c_{t+1}}\left(Y_{t+1}+P_{t+1}\right)\right\} \\ &= E\left\{\beta\frac{Y_t}{Y_{t+1}}\left(Y_{t+1}+P_{t+1}\right)\right\} \\ &\Rightarrow \\ \frac{P_t}{Y_t} &= E\left\{\beta\left(1+\frac{P_{t+1}}{Y_{t+1}}\right)\right\} \end{aligned}$$

We guess that the ratio $\frac{P_t}{Y_t}$ is constant for all t, i.e., $\bar{P} = \frac{P_t}{Y_t}$. This implies

$$\begin{aligned} \bar{P} &= E\left\{\beta\left(1+\bar{P}\right)\right\} \\ &\Rightarrow \\ \bar{P} &= \frac{\beta}{1-\beta} \end{aligned}$$

3. Consider a risky one-period asset which gives a claim to GDP next period, Y_{t+1} (and nothing thereafter). The asset is in zero net supply (so the return on the asset does not affect GDP). Derive the equilibrium condition for the price of this asset and solve for the price.

   Answer:

$$p_t = E\left\{\beta\frac{Y_t}{Y_{t+1}}Y_{t+1}\right\} = \beta Y_t$$

4. Suppose a one-period bond in zero net supply is introduced in the economy. The bond pays one unit of consumption next period. Derive the equilibrium condition for the price of the bond and solve for this price.

   Answer:

$$\begin{aligned} q_t &= E\left\{\beta\frac{Y_t}{Y_{t+1}}\right\} = \beta E\left\{\exp\left(-\varepsilon_{t+1}\right)\right\} \\ &= \beta\exp\left(\frac{\sigma^2}{2}+\frac{\sigma^2}{2}\right) = \beta\exp\left(\sigma^2\right) \end{aligned}$$

5. Explain why the risky asset yields a higher expected return than the bond. Give the intuition for this result.

   Answer: return on bond is $1/q = \exp\left(-\sigma^2\right)/\beta$. Expected return on risky asset is

$$E_t\frac{Y_{t+1}}{p_t} = \frac{1}{\beta}E_t\frac{Y_{t+1}}{Y_t} = \frac{E_t\exp\left(\varepsilon_{t+1}\right)}{\beta} = \frac{1}{\beta}.$$

   Expected return on risky asset is higher than the return on the bond because the risky asset has a high correlation with consumption growth, while the bond return is uncorrelated with consumption growth. This commands an equity premium.

6. Consider now taking this model to the data. Explain why assuming complete markets would allow us to focus on a representative agent version of the economy and why it is reasonable to let the counterpart to Y_t be aggregate consumption.

   Answer: Preferences are on the Gorman form. Since markets are complete, we therefore have aggregation, so individual consumption c_t is proportional to aggregate consumption C_t. Since there is no investments, the aggregate market-clearing condition implies $C_t = Y_t$. Therefore the empirical measure for aggregate consumption must be the counterpart to Y_t in the model.

7. Explain why this model can be expected to yield quantitatively wrong implications for asset pricing.

   Answer: The equity premium puzzle states that with time-additive preferences, frictionless and complete markets, and aggregation (so that aggregate consumption can be used to price assets), then the model requires an unreasonably large risk aversion in order to account for the equity premium. Moreover, with such large risk aversion, the discount factor β required to match the empirical interest rate (close to zero), becomes unreasonably low. These conditions are satisfied in this example because the risk aversion is unity. Hence, when applying a reasonable stochastic process for consumption, the model implies an unreasonably low equity risk premium and an unreasonably low Sharpe ratio (expected equity premium divided by the standard deviation of the equity premium)

3 Business cycles

Consider a New-Keynesian model. Households have preferences over a final good C_t produced with a continuum of goods: $C_t \equiv \left(\int_0^1 C_t(i)^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$. The producer of each good is a monopolist for this good. Firms are subject to a Calvo price friction. Namely, a random set θ of all firms are forced to keep their price fixed at the same level as last year, while the rest of the $1-\theta$ of firms are free to choose the current price of their good. Suppose the economy is in steady state in period $t = 0$.

1. Suppose the economy is hit by a temporary demand shock, where consumers get a low discount factor for one period. Namely, they discount next period utility by a factor $\tilde{\beta}$, where $\tilde{\beta} < \beta$, and next period they return to the regular discount factor β. Explain the dynamics of GDP, hours worked, consumption, and inflation following this shock. In particular, emphasize the intuition for the effects.

   Answer: Assume that monetary policy does not react to the change in β, so the interest rate remains unchanged. With a constant interest rate and

lower β, the Euler equation increases current consumption. The temporary increase in demand for consumption induces the firms with flexible prices to increase their price slightly (recognizing that demand will be back to normal next period), their demand for workers, and their production. This increases inflation slightly. Firms with fixed prices will increase their production and demand for workers in order to meet the increased demand (due to lower β and positive inflation). In the aggregate, there is a positive shift in inflation, labor supply, and GDP.

2. Suppose the economy is hit by a transitory productivity shock, where TFP is $Z_1 = \bar{Z} + \varepsilon$ in the initial period and $Z_t = \bar{Z}$ thereafter. Explain the dynamics of GDP, hours worked, consumption, and inflation following this shock. In particular, emphasize the intuition for the effects.

   Answer: The temporarily high TFP has differential effects on firms that can change the prices and firms that cannot change. Those with flexible prices lower their prices, increase employment, and increase production. This creates deflation (since other firms have unchanged prices). Those with rigid prices lower their production (due to deflation) and their demand for workers (due to the TFP shock and due to deflation). The net effect is an aggregate deflation and an aggregate decline in employment, with an ambiguous effect on aggregate production. However, in the New-Keynesian model, the output gap will become negative

3. How would the economy react to a TFP shock if prices instead were flexible?

   Answer: With flexible prices the economy behaves as an RBC economy. The positive TFP shock will induce firms to lower their prices (causing deflation) and increase production and demand for workers. Due to temporarily high wages, the workers increase their labor supply.