

Exam in ECON5300/9300B

November 25, 2020

1 Real business cycle theory (40%)

Consider a stochastic neoclassical growth model with endogenous labor supply h_t where preferences of a representative household are given by

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where β is the discount factor and c_t is consumption. Production of a standard consumption good takes place with firms renting capital and labor from households at competitive markets. The economy is closed and aggregate production is given by

$$Y_t = Z_t (K_t)^\alpha (H_t)^{1-\alpha},$$

where GDP where K_t and H_t are aggregate values for capital and labor supply, respectively. Z_t is aggregate TFP. The capital stock moves according to $K_{t+1} = (1 - \delta) K_t + I_t$, where I_t is aggregate investment.

1. Assume first that hourly wage w_t grows at a constant rate $g > 0$ so that the wage rate in period t is given by $w_t = (1 + g)^t w_0$.

Question: Explain why preferences of the type $u(c, h) = \log c - v(h)$ or $u(c, h) = \left(c^\theta (1 - h)^{1-\theta} \right)^{1-\gamma} / (1 - \gamma)$ is a necessary condition in order for the economy to have a balanced growth path (steady state).

2. Assume that the preferences of a representative household is given by $u(c, h) = \log c - h^{1+\phi} / (1 + \phi)$.

Question: Using the first-order conditions for the households of this economy, explain how labor supply reacts to shocks to Z_t . What is the Frisch elasticity of labor supply?

3. Suppose that Z_t is an AR(1) process, i.e.,

$$\log Z_t = \rho \log Z_{t-1} + \varepsilon_t.$$

Question: Explain why TFP shocks ε_t have a larger propagation (i.e., impact in terms of magnifying the business cycle fluctuations) in this economy if ρ is closer to zero than if ρ is larger.

4. In the model above, households can choose freely their desired labor supply H_t . Suppose now instead that people can work either zero hours or “full time” \bar{h} , i.e., $h_t \in \{0, \bar{h}\}$.

Questions:

- (a) Write down a social planner problem where the planner chooses each period how many individuals should work $h_t = \bar{h}$ and how many should work zero hours.
- (b) Show that the planner problem can be reformulated as a representative household with (possibly) different preferences from the individuals in the economy.
- (c) What is the aggregate elasticity of labor supply? Explain why it is larger than the elasticity at the individual level (along the intensive margin).
- (d) Show that the optimal allocation in the planner problem can be decentralized with the use of employment lotteries each period, where individuals purchase lottery tickets that determine the probability that they must work this period and the wage and unemployment benefit they get if they end up as employed and unemployed, respectively.
- (e) Discuss verbally how the allocations would change if markets were incomplete (no insurance against unemployment). How would this change affect the answer to 5c above?

2 Asset pricing (60%)

Consider an economy with a representative agent with preferences

$$U_0 = \sum_{t=0}^{\infty} \beta^t u(c_t), \quad (1)$$

The representative household owns a tree that yields one unit of fruit of the consumption good c every period. This fruit cannot be stored between periods. Assume first that the economy is closed and that the household does not have any other sources of income.

1. **Question:** define a competitive equilibrium and calculate the return on bonds, the price of the tree (after picking the fruit), and the consumption path in equilibrium.
2. Suppose that in period $t = 0$ the households experience an unexpected preference shock where they temporarily become more patient: the one

period ahead discount factor increases temporarily to unity and then returns to its initial value $\beta < 1$. Namely, in period $t = 0$ the preferences are

$$U_0 = u(c_0) + \sum_{t=1}^{\infty} \beta^{t-1} u(c_t),$$

and from period $t = 1$ and onward the preference are back to normal as in equation (1).

Question: Show how prices and allocations behave over time in equilibrium.

3. Assume instead that the economy is a small open economy where the households can purchase bonds at the world market. Moreover assume that the real return on one-period bonds is $1 + r = 1/\beta$. The household can also trade in claims to fruits on the tree. Assume further that the period utility function is CRRA with risk aversion γ ,

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

Suppose now that in period $t = 0$ the one-period interest rate (on bonds paying off in period $t = 1$) suddenly falls to zero. But after one period, the interest rate r_t returns to $1/\beta - 1$. However, different from question 2 above, the discount factor for the household on “our” island remains constant at β (thus, the reason for the change in r_t is unrelated to our household).

Question: What is the paths for optimal consumption of the household and the equilibrium price of the tree in this economy?

4. Assume instead that in period $t = 0$ it is announced that the world-market interest rate will remain at zero for 10 periods, and return to $1 + r_t = 1/\beta$ after 10 periods (we maintain the assumption that the household’s discount factor remains constant, so the changes in r_t are unrelated to the household’s discount factor).

Question: What is the evolution of the price of the asset, the optimal consumption, and the ownership of the tree? You may illustrate your answer by drawing qualitative graphs

5. Norway has an oil fund and a fiscal rule (“Handlingsregelen”) which dictates how much the government extracts from the fund. Until 2017 the rule was to spend a fixed fraction 4% of the value of the fund. In 2017 the rule was modified to spend 3% of the fund. The stated motivation for this change was that the average return on the fund was expected to be lower (reflecting the fall in the world-market interest rates after the 2008-2009 financial crisis).

Question A: Please discuss this change in fiscal rule in light of economic theory.

Question B: One proposal for a reform of the fiscal rule has been to just take out dividends. Would this be a good alternative rule in the presence of changes in the world-market interest rate?