ECON5300/9300B - Fall 2022

The exam consists of three parts: A, B, C, and D. All parts have the same weight (25%).

Part A: Gorman form and incomplete markets (25%)

A household solves the following problem:

$$\max_{c_1,c_2} u(c_1) + u(c_2)$$

subject to

$$c_1 + a = y_1$$

$$c_2 = a + y_2$$

a) Show that the consumption functions satisfy the Gorman form if $u(c) = \log c$.

b) Assume now that there is a borrowing constraint, $a \geq 0$. Solve for c_1 and c_2 as functions of y_1 and y_2.

c) Draw the solution for c_1 in a) and b). Explain why the consumption function in b) does not satisfy the Gorman form.

d) Under which assumption does there exist a representative agent in problem b), i.e., where the behavior of a single agent is representative of the population.

Part B: Neoclassical Growth Model (25%)

We are first going to solve the Neoclassical Growth model. The model is

$$\max_{[k(t),c(t)]_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} u(c(t))dt$$

$$\text{subject to}$$

$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t)$$

a) Compute the Euler-equation.

b) Compute the steady state level of capital and consumption with calibration $\delta = 0.05$, $\rho = 0.05$, and $f(k(t)) = k(t)^{\alpha}$ with $\alpha = 0.3$.

c) Compute the golden rule level of capital k^g. Explain what the golden rule level of capital is and why k^* is lower than k^g in the calibration in b?

Part C: OLG Model (25%)

The household problem in the OLG model is

$$\max_{c_1(t),c_2(t+1)} \frac{c_1(t)^{1-\theta}}{1-\theta} + \beta\frac{c_2(t+1)^{1-\theta}}{1-\theta}$$

subject to

$$c_1(t) + s(t) = w(t)$$

$$c_2(t+1) = R(t+1)s(t)$$

a) Solve for $s(t)$ as a function of $w(t)$.

b) Aggregate per-capita savings is defined as $k(t+1) = \frac{s(t)}{1+n}$ where n is the population growth rate. Moreover, assume that $R(t+1) = f'(k(t)) = \alpha k(t)^{\alpha-1}$, $w(t) = f(k) - f'(k)k = (1-\alpha)k(t)^{\alpha}$, and $\theta \to \infty$. Show that the steady state level of capital is implicitly defined as

$$1 = \frac{(1-\alpha)f'(k)}{\alpha(1+n)}\frac{1}{f'(k)+1}.$$

c) The social planner solution is $f'(k^{sp}) = 1 + n$. Explain what dynamic inefficiency is and what policies can be implemented to address it.

Part D: The New-Keynesian Model (25 %)

We will use the New Keynesian model to see how the central bank should optimally respond to a cost-push shock. We assume that the model is

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\begin{aligned}
\pi_t &= \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa x_t + u_t \\
x_t &= \mathbb{E}_t\{x_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\}) \\
u_t &= \rho u_{t-1} + \varepsilon_t \\
\varepsilon_t &\sim N(0, \sigma)
\end{aligned}
$$

where π is inflation, x is the output gap, i is the nominal interest rate, and u is a cost-push shock. $\kappa > 0$, $\rho > 0$, and $\sigma > 0$ are known parameters. Assume that the central bank's loss function is $L_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [\pi_{t+k}^2 + \lambda x_{t+k}^2]$.

a) Solve for the optimal policy under discretion and interpret the first-order conditions.

b) Use the method of undetermined coefficients to solve for the output and inflation response to the cost-push shock under optimal monetary policy.
Hint: assume the output and inflation responses are linear in the cost-push shock.

c) The real interest rate is defined as $r_t = i_t - \mathbb{E}_t\{\pi_{t+1}\}$. Solve for the optimal real interest rate response to the cost-push shock.

d) The situation today with high inflation can be interpreted as a large positive cost-push shock. The solution in c) says that the real interest rate should *increase* in response to a cost-push shock. However, the real interest rate in Norway is still negative. Discuss briefly why the central bank may want to deviate from the optimal policy prescription of this model.