The exam consists of four parts: A, B, C, and D. All parts have the same weight (25%).

Part A: Gorman form and incomplete markets (25%)

A household solves the following problem:

$$\max_{c_1,c_2} u(c_1) + u(c_2)$$

$$\text{subject to}$$

$$c_1 + a = y_1$$

$$c_2 = a + y_2$$

a) Show that the consumption functions satisfy the Gorman form if $u(c) = \log c$.

   Solution: the Marshallian demand functions are $c_1 = c_2 = \frac{1}{2}(y_1 + y_2)$.

b) Assume now that there is a borrowing constraint, $a \geq 0$. Solve for c_1 and c_2 as functions of y_1 and y_2.

   Solution:

$$c_1 = \begin{cases} \frac{1}{2}(y_1 + y_2) & \text{if } y_1 \geq y_2 \\ y_1 & \text{if } y_1 < y_2 \end{cases}$$

$$c_1 = \begin{cases} \frac{1}{2}(y_1 + y_2) & \text{if } y_1 \geq y_2 \\ y_2 & \text{if } y_1 < y_2 \end{cases}$$

c) Draw the solution for c_1 in a) and b). Explain why the consumption function in b) does not satisfy the Gorman form.

   Solution: the consumption function in b) is kinked. Redistribution between agents above and below the kink leads to changes in aggregate consumption.

d) Under which assumption does there exist a representative agent in problem b), i.e., where the behavior of a single agent is representative of the population.

   Solution: if all agents have $y_1 < y_2$ *or* $y_1 \geq y_2$*, the economy admits a representative agent because every agents' consumption function slopes are the same.*

Part B: Neoclassical Growth Model (25%)

We are first going to solve the Neoclassical Growth model. The model is

$$\max_{[k(t),c(t)]_{t=0}^{\infty}} \int_0^{\infty} e^{-\rho t} u(c(t)) dt$$

subject to

$$\dot{k}(t) = f(k(t)) - \delta k(t) - c(t)$$

a) Compute the Euler-equation.

Solution: $\frac{\dot{c}(t)}{c(t)} = f'(k(t)) - \delta - \rho$

b) Compute the steady state level of capital and consumption with calibration $\delta = 0.05$, $\rho = 0.05$, and $f(k(t)) = k(t)^{\alpha}$ with $\alpha = 0.3$.

Solution: $k^* = \left(\frac{\alpha}{\delta+\rho}\right)^{1/(1-\alpha)} = (0.3/0.1)^2 = 9$; $c^* = (k^*)^{\alpha} - \delta k^* = \sqrt{9} - 0.05 * 9 = 2.55.$

c) Compute the golden rule level of capital k^g. Explain what the golden rule level of capital is and why k^* is lower than k^g in the calibration in b?

Solution: golden rule capital in this model is $f'(k^g) = \delta$, *which implies* $k^g = \left(\frac{\alpha}{\delta}\right)^{1/(1-\alpha)} = (0.3/0.05)^2 = 36$. *The golden rule level of capital is the level of capital that maximizes steady state consumption. The golden-rule level of capital, however, implies that the interest rate is 0, which in this calibration is lower than the strictly positive discount rate. Such a high level of capital is therefore not optimal because it induces households to dissave because the discounted marginal utility from saving one more unit is higher than the marginal utility of consuming today.*

Part C: OLG Model (25%)

The household problem in the OLG model is

$$\max_{c_1(t),c_2(t+1)} \frac{c_1(t)^{1-\theta}}{1-\theta} + \beta\frac{c_2(t+1)^{1-\theta}}{1-\theta}$$

subject to

$$c_1(t) + s(t) = w(t)$$
$$c_2(t+1) = R(t+1)s(t)$$

a) Solve for $s(t)$ as a function of $w(t)$.

Solution: $s(t) = w(t)\frac{\beta^{1/\theta}R(t+1)^{1/\theta-1}}{1+\beta^{1/\theta}R(t+1)^{1/\theta-1}}$.

b) Aggregate per-capita savings is defined as $k(t+1) = \frac{s(t)}{1+n}$ where n is the population growth rate. Moreover, assume that $R(t+1) = f'(k(t)) = \alpha k(t)^{\alpha-1}$, $w(t) = f(k) - f'(k)k = (1-\alpha)k(t)^{\alpha}$, and $\theta \to \infty$. Show that the steady state level of capital is implicitly defined by

$$1 = \frac{(1-\alpha)f'(k)}{\alpha(1+n)}\frac{1}{f'(k)+1}.$$

Solution: inserting into the definition of aggregate savings:

$$k = \frac{s}{1+n} = \frac{w}{1+n}\frac{\beta^{1/\theta}f'(k)^{1/\theta-1}}{1+\beta^{1/\theta}f'(k)^{1/\theta-1}} = \frac{f(k)-f'(k)k}{1+n}\frac{\beta^{1/\theta}f'(k)^{1/\theta-1}}{1+\beta^{1/\theta}f'(k)^{1/\theta-1}}$$

which under the limit ($\theta \to \infty$) becomes

$$\lim_{\theta\to\infty} k = \lim_{\theta\to\infty}\frac{f(k)-f'(k)k}{1+n}\frac{\beta^{1/\theta}f'(k)^{1/\theta-1}}{1+\beta^{1/\theta}f'(k)^{1/\theta-1}} = \frac{f(k)-f'(k)k}{1+n}\frac{1}{f'(k)+1}$$

which implies that

$$1 = \frac{f(k)/k - f'(k)}{1+n}\frac{1}{f'(k)+1} = \frac{f'(k)/\alpha - f'(k)}{1+n}\frac{1}{f'(k)+1} = \frac{(1-\alpha)f'(k)}{\alpha(1+n)}\frac{1}{f'(k)+1}.$$

c) The social planner solution is $f'(k^{sp}) = 1+n$. Explain what dynamic inefficiency is and what policies can be implemented to address it.

Solution: Dynamic inefficiency happens when $f'(k) < 1+n$. Dynamic inefficiency is a situation with overaccumulation of capital because households have very stong lifecycle *motives for saving. The market rate will be too low because in this economy, there exists an alternative "population" rate of return which the social planner has access to (e.g., pay-as-you-go pension system or government debt).*

Part D: The New-Keynesian Model (25 %)

We will use the New Keynesian model to see how the central bank should optimally respond to a cost-push shock. We assume that the model is

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa x_t + u_t$$
$$x_t = \mathbb{E}_t\{x_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\})$$
$$u_t = \rho u_{t-1} + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma)$$

where π is inflation, x is the output gap, i is the nominal interest rate, and u is a cost-push shock. $\kappa > 0$, $\rho > 0$, and $\sigma > 0$ are known parameters. Assume that the central bank's loss function is $L_t = \mathbb{E}_t \sum_{k=0}^{\infty} \beta^k [\pi_{t+k}^2 + \lambda x_{t+k}^2]$.

a) Solve for the optimal policy under discretion and interpret the first-order conditions.

Solution: optimal policy under discretion implies $\lambda x_t = -\kappa \pi_t$. Marginal cost of leaning in terms of output deviations equals marginal benefits in terms of reducing inflation deviations.

b) Use the method of undetermined coefficients to solve for the output and inflation response to the cost-push shock under optimal monetary policy.
Hint: assume the output and inflation responses are linear in the cost-push shock.

Solution: assume $x_t = \psi_{xu} u_t$ and $\pi_t = \psi_{\pi u} u_t$. Inserting these assumptions into the Phillips curve and the leaning against the wind condition gives:

$$\pi_t = \frac{\lambda}{\kappa^2 + \lambda(1 - \beta\rho)} u_t$$
$$x_t = -\frac{\kappa}{\kappa^2 + \lambda(1 - \beta\rho)} u_t$$

c) The real interest rate is defined as $r_t = i_t - \mathbb{E}_t\{\pi_{t+1}\}$. Solve for the optimal real interest rate response to the cost-push shock.

Solution: the DIS implies that $r_t = \mathbb{E}_t\{x_{t+1}\} - x_t$. Insert for the solution for x_t in b and get: $r_t = \frac{\kappa(1-\rho)}{\kappa^2 + \lambda(1-\beta\rho)}$.

d) The situation today with high inflation can be interpreted as a large positive cost-push shock. The solution in c) says that the real interest rate should *increase* in response to a cost-push shock. However, the real interest rate in Norway is still negative. Discuss briefly why the central bank may want to deviate from the optimal policy prescription of this model.

Solution: this is an open question. The student should discuss that in the model above, the only channel through which the central bank can reduce output is to raise the real interest rate. The central bank may be concerned that there are other channels operating (e.g., the cash-flow channel) where an interest rate hike is contractionary even when the real interest rate is negative. Central banks also have alternative mandates that may play a role (e.g., financial stability) and they may use a gradual approach because they know that the interest rate affects the real economy with a delay.