Lecture 5. Incomplete Contracts

So far we have considered models in which the agents face no constraints on the form of contracts they can agree upon: the only limit is observability. The contracts we considered can include prescriptions for every possible instance (or "state of the world"). These contracts are called complete. Most contracts in the real world are incomplete. In most situations, it is virtually impossible, ex ante, to consider explicitly all the possible future events, either because the agents ignore even the possibility of certain events or because there are too many possible instances and there is no contract that can take them all into account. When contracts are incomplete, institutional arrangements such as ownership titles, decision-making rules and authority matter. An ownership title can be seen as the right to dispose of a property as one sees fit, in all situations in which the law or a private contract does not specify what the owner must do of the property. These are called "residual property rights".

Tirole (1999) provides a good example.[1] Patents exists to reward innovation. In a world of complete contracts, innovation could be rewarded with a prize proportional to the social value of the innovation. It is difficult to estimate this value when the innovation is introduced (and also after), and it would be hard to establish a reliable institution providing this prize. Patents are a less-than-socially-efficient way to reward innovation. Patents give monopoly power, but information is a public good and therefore efficiency would require to distribute it freely.

Incomplete contracts have been primarily considered in the context of organizational economics: the study of the determinants of the size of the firm and the allocation of authority within a firm.

Ownership and the Property-Right Theory of the Firm

The reference is BD, section 11.2, and Grossman and Hart (1986).[2]

In a first stage, each agent in a set I makes an investment x_i at cost $\psi(x_i)$. Let $\overline{A}$ be the set of productive assets.

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[1] "Incomplete Contracts: Where Do We Stand?", Econometrica.

[2] "The Costs and Benefits of Ownership: A Theory of Vertical and Lateral Integration," JPE, 1986.

In a second stage, the investments of a subset $S \subseteq I$ of agents, combined with a subset of assets $A \subseteq \overline{A}$, generates a value equal to $V(S; A|x)$, where $x = (x_1, x_2, .., x_I)$.

Assume that the surplus is divided according to the Shapley Value.

Definition. *Let $\mathbb{S}$ denote the set of all possible subsets S and $\mathbb{A}$ the set of all possible subsets A. $\omega : \mathbb{S} \to \mathbb{A}$ denotes the subset of assets owned by each subset of agents.*

$\omega(\cdot)$ has two properties:

1) $\omega(S) \bigcap \omega(I \backslash S) = \emptyset$: an asset can be owned by at most one agent.

2) $S' \subseteq S \to \omega(S') \subseteq \omega(S)$: adding new agents cannot reduce the amount of assets owned by a subset of agents.

Definition. *Given an $\omega(\cdot)$, a vector of ex-ante investments x, and an associated surplus for any given group of agents $V(S, \omega(S)|x)$ the Shapley value specifies the following expected ex-post surplus for any agent i:*

$$B_i(\omega|x) \equiv \sum_{S|i \in S} p(S) \left\{ V\left(S; \omega(S)|x\right) - V\left(S \backslash \{i\}; \omega(S \backslash \{i\})|x\right) \right\}$$

where $p(S) = \frac{(s-1)!(|I|-s)!}{|I|!}$ and $s = |S|$.

$p(S) = Pr($combination of $s-1$ elements from a set of $|I| - 1) Pr(i$ in position $s) = \frac{1}{C(|I|-1, s-1)} \frac{1}{I} = \frac{(|I|-s)!(s-1)!}{(|I|-1)!} \frac{1}{I} = \frac{(|I|-s)!(s-1)!}{|I|!}$.

In the case of two agents, this means that the only way in which ownership affects the final outcome is through the division of ex-post surplus and therefore the incentive to invest in the first stage.

Example 1. Printer-publisher Integration

$I = 2$, (printer and publisher), $\overline{A} = \{a_1, a_2\}$ (publishing business, printing press). Three possible ownership allocations:

No integration: $\omega(1) = \{a_1\}$, $\omega(2) = \{a_2\}$, Agent-1 (printer) integration: $\omega(1) = \{a_1, a_2\}$, $\omega(2) = \{\emptyset\}$,

Agent-2 (publisher) integration: $\omega(1) = \emptyset$, $\omega(2) = \{a_1, a_2\}$.

Assumption 1. *Suppose that no output can be produced unless both assets are used,*

No Integration

By assumption 1: $V(\{1\}; \{a_1\} | x) = V(\{2\}; \{a_2\} | x) = 0$, where $x := \{x_1, x_2\}$.

Let the surplus generated by the publisher and printer together be defined as:

$V(x) := V(\{1, 2\}; \{a_1, a_2\} | x)$.

The Shapley value assigns to each agent an expected payoff:

$B_1(NI|x) = B_2(NI|x) = \frac{1}{2}V(x)$.

Printer Integration

Under printer integration, let the surplus generated by the printer alone be defined as:

$\Phi_1(x_1) := V(\{1\}; \{a_1, a_2\} | x)$, where $\Phi_1(x_1) < V(x)$

By assumption 1: $V(\{2\}; \{\emptyset\} | x) = 0$.

The Shapley value assigns to each agent an expected payoff:

$B_1(I_1|x) = \Phi_1(x_1) + \frac{1}{2}(V(x) - \Phi_1(x_1))$,

$B_2(I_2|x) = \frac{1}{2}(V(x) - \Phi_1(x_1))$.

Publisher Integration

Symmetrically, in this case the Shapley value assigns to each agent an expected payoff:

$B_1(I_1|x) = \frac{1}{2}(V(x) - \Phi_2(x_2))$,

$B_2(I_2|x) = \Phi_2(x_2) + \frac{1}{2}(V(x) - \Phi_2(x_2))$.

Ex-ante investments

Let $V(x)$ be increasing and concave in x, let $\Phi_i(x_i)$ be increasing and concave in x_i, and let $\psi_i(x_i)$ be increasing and convex.[3] Investments are chosen by the agents non-cooperatively in order to maximize their respective expected payoff:

[3] $V(.)$ is a multivariate function. A concave multivariate function $f(x)$ satisfies $f(\lambda x + (1-\lambda)x') \geq \lambda f(x) + (1-\lambda)(x')$, $\forall x, x'$ and $\forall \lambda \in (0, 1)$ where $\lambda x + (1-\lambda)x'$ is a vector addition.

$$\max_{x_i} \left\{ B_i \left[\omega(S) | x \right] - \psi_i(x_i) \right\}$$

If investment levels were chosen cooperatively, property rights would not matter.

Under no integration, investment levels satisfy:

$$\begin{aligned} \frac{1}{2} \frac{\partial V(x_1^{NI}, x_2^{NI})}{\partial x_1} &= \psi_1'(x_1^{NI}), \\ \frac{1}{2} \frac{\partial V(x_1^{NI}, x_2^{NI})}{\partial x_2} &= \psi_2'(x_2^{NI}). \end{aligned}$$

Under printer integration:

$$\begin{aligned} \Phi_1'(x_1^{I1}) + \frac{1}{2} \left(\frac{\partial V(x_1^{I1}, x_2^{I1})}{\partial x_1} - \Phi_1'(x_1^{I1}) \right) &= \psi_1'(x_1^{I1}), \\ \frac{1}{2} \frac{\partial V(x_1^{I1}, x_2^{I1})}{\partial x_2} &= \psi_2'(x_2^{I1}). \end{aligned}$$

And similarly under publisher integration:

$$\begin{aligned} \frac{1}{2} \frac{\partial V(x_1^{I2}, x_2^{I2})}{\partial x_1} &= \psi_1'(x_1^{I2}), \\ \Phi_2'(x_2^{I2}) + \frac{1}{2} \left(\frac{\partial V(x_1^{I2}, x_2^{I2})}{\partial x_2} - \Phi_2'(x_2^{I2}) \right) &= \psi_2'(x_2^{I2}) \end{aligned}$$

For a given x_2, printer's (agent-1) incentives to invest are strongest under printer integration (as long as $\Phi_1'(x_1) > 0$), and symmetrically, for a given x_1, publisher's (agent-2) incentives to invest are strongest under publisher integration (as long as $\Phi_2'(x_2) > 0$). If, however, $\Phi_1'(x_1) < 0$ (or $\Phi_2'(x_2) < 0$) integration might have no effect or a negative effect on investment. Note that

$\Phi_i'(x_i) \leq 0$ is not only a theoretical curiosity: think of investments that are very specific to the technology used by the other agent (customization).

Equilibrium Ownership Structures

As long as agents are endowed with enough resources to trade property rights ex ante and there are no other barriers or costs to trade, we should expect the ex-ante efficient allocation to arise in equilibrium. In order to think about what conditions make each allocation the equilibrium one, let's first consider the socially efficient levels of investment (x_1^*, x_2^*). These levels satisfy:

$$\frac{\partial V(x_1^*, x_2^*)}{\partial x_1} = \psi_1'(x_1^*),$$
$$\frac{\partial V(x_1^*, x_2^*)}{\partial x_2} = \psi_2'(x_2^*).$$

Let the two parties investments be complementary, that is $\frac{\partial^2 V(x_1,x_2)}{\partial x_2 \partial x_1} \geq 0$. Then:

1) If $\Phi_i'(x_i) > 0$, under $i-$integration both agents invest more than under no integration.

2) If instead $\Phi_i'(x_i) \leq 0$, no integration results in (weakly) higher investments than integration.

Note that for positive large $\Phi_i'(x_i)$ investment levels under integration could be above the socially efficient levels, so there can be over investment under integration. If $0 \leq \Phi_i'(x_i) \leq \frac{\partial V(x_i,x_j)}{\partial x_i}$ $\forall x_j$, then integration dominates no integration.

Overall message: some kind of integration is optimal if marginal returns on investment are highest when all assets and agent are combined ex post and when the marginal return on investment remains positive even if the owner of all assets does not hire the other agent $(\Phi_i'(x_i) > 0)$.

Hart and Moore (1990) extend the analysis.[4] They consider the case of many agents, and agents that do not own any asset, but are necessary for production. While Grossman and Hart (1986) is a model of top management, Hart and Moore (1990) can be thought of as a model of employers and employees. A numerical example provides the intuition of their results.

[4]"Property Rights and the Nature of the Firm", JPE.

3 agents: skipper, chef and tycoon, 1 asset: a yacht. The payoff is shared between agents following the Shapley Value.

Case 1

The service is worth 240 at time 1 to the tycoon if the chef takes an investment at time 0. The investment costs 100 to the chef and is not transferable to other yachts. The skipper can be replaced at time 1 at no cost.

If the skipper owns the yacht, the chef does not invest, because the expected payoff from investing is:

$\frac{1}{3}240 - 100 = -20 < 0$.

If the chef owns the yacht, she invests because:

$\frac{1}{3}\frac{1}{2}240 + \frac{1}{3}240 - 100 = 20 > 0$.

If the tycoon owns the yacht, the chef invests:

$\frac{1}{3}\frac{1}{2}240 + \frac{1}{3}240 - 100 = 20 > 0$.

The chef is more likely to invest in a skill that is tycoon-specific if the asset is owned by the chef or by the tycoon.

→ *Even if an agent does not own the asset, it matters to her who owns it.*

Case 2

Both the skipper and the chef can take an investment at cost 100 and each investment increases the value by 240. Both the skipper and the chef can be replaced on the market (but the replacements have not invested).

If the skipper owns the yacht, the skipper invests (I) and the chef does not invest (II).

(I): $\frac{1}{3}\frac{1}{2}240 + \frac{1}{3}240 - 100 = 20 > 0$,

(II): $\frac{1}{3}240 - 100 = -20 < 0$.

For the same reason, if the chef owns the yacht, the chef invests and the skipper does not. If the tycoon owns the yacht, both chef and skipper invest as:

$\frac{1}{3}\frac{1}{2}240 + \frac{1}{3}(480 - 240) - 100 = 20 > 0$.

→ *If an agent is indispensable, it is efficient to assign to the property right to her, regardless of whether she has the option to take an investment ex-ante.*

Case 3

Each agent can take an investment that increases the value by 240, and costs c_1, c_2 and c_3 respectively (let 1 refer to the skipper, and 2 to the chef). Assume that all three agents can be replaced in period 1. Assume also that ownership of the yacht can be shared between chef and skipper (so that they are both necessary for the yacht to be available).

If ownership is shared, investments take place if:

$\frac{1}{3}\frac{1}{2}480 + \frac{1}{3}720 - c_1 \geq \frac{1}{3}480 + \frac{1}{3}\frac{1}{2}240 \leftrightarrow c_1 \leq \frac{1}{2}240$,

$c_2 \leq \frac{1}{2}240$,

$c_3 \leq \frac{1}{3}240$.

If the skipper owns the entire yacht, the following conditions are sufficient

$\frac{1}{3}240 + \frac{1}{3}240 + \frac{1}{3}240 - c_1 \geq 0 \leftrightarrow c_1 \leq 240$

$\frac{1}{3}\frac{1}{2}240 + \frac{1}{3}720 - c_2 \geq \frac{1}{3}480 \leftrightarrow c_2 \leq \frac{1}{2}240$,

$c_3 \leq \frac{1}{3}240 + \frac{1}{3}\frac{1}{2}240 = \frac{1}{2}240$.

If property is entirely in the hands of the skipper, both skipper and tycoon are more likely to invest, while the chef is as likely to invest as under separate ownership.

→ *if assets are very complementary, it is efficient to concentrate ownership.*