Sensorveiledning MAE4011 - Principles of measurement

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Task 9

$$\hat{\mathrm{Cor}}(X,Y) = \frac{\hat{\mathrm{Cov}}(X,Y)}{\hat{\sigma_X}\hat{\sigma_Y}} = \frac{-7}{\sqrt{10}\times\sqrt{10}} = \frac{-7}{10} = \frac{-7}{10} = -0.7.$$

i Identify correct components to use to calculate the correlation. (+0.5p)

ii Calculate it correctly. (+0.5p)

iii Answer the question correctly based on the result, i.e. the relationship is moderately to strongly negative. (+0.5p)

iv Either of the following adds another 0.5p:

- Comment that the correlation measures a linear relationship.
- Comment on the sampling variance being a potential factor which adds uncertainty to the estimate.
- Comment on measurement error influencing the result by decreasing the magnitude of the relationship.
- Other things that make sense.

Task 10

Only the answers are required, 0.5p each. This item is automatically scored but needs a quick check to ensure the scoring was done right.

a

The mean is

$$\begin{aligned}E(X) &= P(X=0)\times 0 + P(X=1)\times 1 + P(X=2)\times 2\\ &= 0.2\times 0 + 0.6\times 1 + 0.2\times 2\\ &= 0 + 0.6 + 0.4\\ &= 1.\end{aligned}$$

b

The mode is 1.

c

The median is 1.

d

We have that $\mathrm{Var}(X) = E(X^2) - (E(X))^2$.

$$\begin{aligned}E(X^2) &= P(X=0)\times 0^2 + P(X=1)\times 1^2 + P(X=2)\times 2^2\\ &= 0.2\times 0^2 + 0.6\times 1^2 + 0.2\times 2^2\\ &= 0 + 0.6 + 0.8\\ &= 1.4.\end{aligned}$$

Hence,

$$\mathrm{Var}(X) = 1.4 - 1^2 = 1.4 - 1 = 0.4.$$

An interval 0.3-0.5 is acceptable.

Task 11

a

$$\mathrm{Var}(X-Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) - 2\mathrm{Cov}(X,Y) = 2 + 3 - 2\times 1 = 3$$

For full marks (1p), a correct presentation or explanation of the computation is required. Only the final answer gives 0.5p.

b

$$\begin{aligned}\mathrm{Var}(X+2Y) &= \mathrm{Var}(X) + \mathrm{Var}(2Y) + 2\mathrm{Cov}(X,2Y)\\ &= \mathrm{Var}(X) + 2^2\mathrm{Var}(Y) + 2\times 2\mathrm{Cov}(X,Y)\\ &= 2 + 4\times 3 + 4\times 1\\ &= 18\end{aligned}$$

For full marks (1p), a correct presentation or explanation of the computation is required. Only the final answer gives 0.5p.

Task 12

We then have that

$$\alpha = 5 \times \frac{1^2}{10} = \frac{5}{10} = 0.5.$$

Coefficient alpha is the reliability coefficient of the sum scores under the assumption of a single factor model and equal factor loadings. If only assuming a single factor model, coefficient alpha is always less than the reliability coefficient for the sum scores.

Correct computation gives +1p. Correct interpretation and statement of assumptions, irrespective of correct computation, gives +1p.

Task 13

i Too much: E.g., anxiety, illness, on-site disturbance (loud environment), cheating. (Max 1p.)

ii Too little: E.g., insufficient content coverage, too easy or too difficult questions for specific skills. (Max 1p.)

If listing multiple examples, only give full credits if all examples are appropriate.

Task 14

We plug in the cut score value 30 in the equating function, yielding

$$30 = 1.2X + 6 \iff 24 = 1.2X \iff X = 20.$$

Thus, based on the equating function $eq(Y) = 1.2X + 6$, the cut score for test X should be set to 20.

Correct answer +1p. Correct presentation of the computation +1p.

Task 15

Since the model fits well according to the cut-off values we have discussed in the course (GFI > 0.95, RMSEA < 0.06, SRMR < 0.08), the estimated factor correlation can reasonably be interpreted as the correlation between the underlying factors. The correlation between the sum scores should be lower than the factor correlation due to measurement error.

Full marks should outline that measurement error for the sum scores implies that the correlation is lower than the factor correlation and include a comment on the model fit being important for the interpretation.

Task 16

The bifactor model can be used to assess if a single factor model is approximately appropriate. If a general factor dominates (explains substantially more variance than the specific factors do), then a single factor model can be judged as appropriate. We can compare the variance explained by the general factor, which is

$$\sum_{j=1}^{6} \lambda_{Gj}^2 = 3^2 + 1^2 + 2^2 + 1^2 + 1^2 + 1^2 = 9 + 1 + 4 + 1 + 1 + 1 = 17$$

to the variance explained by the all factors, which is

$$\sum_{j=1}^{6} \lambda_{Gj}^2 + \sum_{j=1}^{6} \lambda_{Sj}^2 = 17 + 0.25 + 0.25 + 1 + 1 + 0.25 + 0.25 = 20.$$

This yields 17 / 20 = 0.85, which is higher than the lowest recommended threshold (0.7). Hence, using the sum score is well-justified based on the small amount of multidimensionality that is present.

+1p to identify the variance explained by the common factor as a measure of the appropriateness of a single factor model, +1p for appropriate computation and conclusion. Other, less precise explanations that reflect the same aspect can also give partial credit up to 1.5p in total.

Task 17

Required components for acceptable responses:

i Identify a suitable approach, e.g. the bookmarking method and briefly describe the procedure. (Max +2p)

ii Suggest a procedure for sorting items by difficulty. (Max +1p)

iii Outline how an expert panel should judge the appropriate cut-score. (Max +1p)

Task 18

i Evidence sources: internal structure, relationship to other variables, content of items (up to +2p)

ii Description of data: random sample from the target population, give the scale to these individuals, record gender of respondents, expert panel to assess items based on content (up to +2p)

iii Results: unidimensional model in accordance with the theoretical framework, differences in factor means with respect to gender, expert evaluation consistent with theory, sufficiently reliable scores (up to +2p)

Task 19

1.

Evidence based on internal structure. (1p)

2.

The model fits well, since the residual correlation matrix does not have any values which are larger than 0.1 in magnitude. (1p)

3.

The information, λ_j^2/Ψ_j^2, is the most appropriate method to evaluate how much the items contribute to the precision. We have information 1, 4.5, 0.25, 0.80 and 4 for items 1 to 5. Hence the rank-ordering is Item 2, Item 5, Item 1, Item 4 and lastly Item 3.

Compute the information of the items and rank-order the items correctly, with justification (1p). Compute another relevant statistic with appropriate motivation (0.5p).

4.

For example that 4-category items should be treated as ordinal. Other reservations that make sense also give credits. If stating more than two reservations, only give full credits if they are all correct. (1p)

Task 20

1

$$X_3 = \lambda_{M3}M + \lambda_{R3}R + E_3$$

M is the math literacy factor, R the reading literacy factor and E_3 is the error term, all of which are random variables. λ_{M3} is the factor loading for math literacy, λ_{R3} is the factor loading for reading literacy and $\Psi_3^2 = \text{Var}(E_3)$ is the variance of the error term, all of which are parameters of the model. λ_{M3} and λ_{R3} says how sensitive the item is at measuring the latent factors M and R and Ψ_3^2 indicates how much random error there exists for item 3.

Correct equation +1p, correct interpretation of all components +1p.

b

We can first note that

$$X_3 = \lambda_{M3}M + \lambda_{R3}R + E_3$$

and

$$X_4 = \lambda_{R4}R + E_4.$$

We then have that

$$\begin{aligned}
\text{Cov}(X_3, X_4) &= \text{Cov}(\lambda_{M3}M + \lambda_{R3}R + E_3, \lambda_{R4}R + E_4) \\
&= \text{Cov}(\lambda_{M3}M, \lambda_{R4}R + E_4) + \text{Cov}(\lambda_{R3}R, \lambda_{R4}R + E_4) + \text{Cov}(E_3, \lambda_{R4}R + E_4) \\
&= \text{Cov}(\lambda_{M3}M, \lambda_{R4}R) + 0 + \text{Cov}(\lambda_{R3}R, \lambda_{R4}R) + 0 + 0 + 0 \\
&= \lambda_{M3}\lambda_{R4}\text{Cov}(M, R) + \lambda_{R3}\lambda_{R4}\text{Var}(R, R).
\end{aligned}$$

We plug in the values for the factor loadings and factor covariances and obtain

$$\mathrm{Cov}(X_3, X_4) = 0.5 \times 0.5 \times 0.8 + 0.5 \times 0.5 \times 1 = 0.2 + 0.25 = 0.45.$$

Entirely correct derivation gives 2p, even if there is some mistake in the computation. Otherwise, 0.5p-1.5p depending on level of completion.