

i General information

Some guidelines:

- The exam consists of a combination of multiple choice questions and free-response questions. Note that the multiple choice questions have only one category which is the correct one.
- When answering questions that ask you to interpret the results, you can obtain partial credit even if you did not calculate the quantities correctly. As long as your interpretation is consistent with your calculations you will obtain at least partial credit.
- Read the questions carefully and write down if you do not understand something about a question. The responsible teacher will visit the exam site for a short period during the first two hours of the exam.

1 Level of measurement 1

An interior design company is interested in the preferences for kitchens among young Norwegian adults and conducts a survey including the following two questions:

I Which out of the following colours do you most prefer for your kitchen?

- White
- Grey
- Brown

II How much are you willing to pay for a new kitchen?

- Less than 50,000 NOK.
- Between 50,000 and 150,000 NOK.
- More than 150,000 NOK.

a) What is the level of measurement for question I?

Select one alternative

- Nominal
- Ordinal
- Interval
- Ratio

b) What is the level of measurement for question II?

Select one alternative

- Nominal
- Ordinal
- Interval
- Ratio

Maximum marks: 2

2 Parameter 1

Which of the following is not a parameter?

Select one alternative:

- A factor loading.
- The sample mean.
- The variance.
- Coefficient alpha.

Maximum marks: 2

3 Variance 1

The variance of a random variable X is 10. What is the variance of the random variable $5X$?

Select one alternative:

- 250
- 500
- 50
- 10

Maximum marks: 2

4 Classical test theory 1

Which of the following statements is always true for the classical true score model?

Select one alternative:

- The observed score is the expected value of the true score.
- The error score is normally distributed.
- The variance of the observed score is the sum of the variance of the true score and the error score.
- Lower true score variance means higher reliability.

Maximum marks: 2

5 Coefficient alpha 1

Based on the information given, under which circumstance can you say that the coefficient alpha is equal to the reliability of the sum score?

Select one alternative:

- When a single factor model with all factor loadings equal is the true model.
- When the true model is a bifactor model with a dominating general factor.
- When the true scores are uncorrelated with the error scores.
- When a single factor model with all items having the same difficulty is the true model.

Maximum marks: 2

6 Validity 1

You have fitted a single factor model to data from a psychological scale and observe the results. What would be evidence for the validity of the sum score as an indicator of a single attribute?

Select one alternative:

- High factor loadings for several items.
- That the factor loadings are all similar.
- That the model fits the data well.
- A high coefficient omega.

Maximum marks: 2

7 Factor model 1

Consider the factor model

$$X_j = \mu_j + \lambda_j F + E_j,$$

where F has variance 1. The estimated parameters for an item were $\mu_j = 3.4$, $\lambda_j = 0.5$ and $\Psi_j^2 = 1.5$. What is the variance of X_j according to the estimated parameters?

Select one alternative:

- 2
- 1.5
- 5.4
- 1.75

Maximum marks: 2

8 Item bias 1

Assume that a single factor model is appropriate for the analysis of several items. Which of the following is evidence of item bias?

Select one alternative:

- The mean item score for individuals with the same sum score is different for men and women.
- The item difficulty is different for two separately estimated factor models for men and women.
- In a linear regression model with the item score as an independent variable, the regression coefficient for the item score is different between men and women.
- The factor loadings are different for two separately estimated factor models for men and women.

Maximum marks: 2

9 Equating 1

For which of the following settings is equating necessary?

Select one alternative:

- In a high-stakes college entrance exam given at one point in time where the score is valid only for that time point.
- In a national exam of mathematical literacy given to students in grade 9, with purpose to assign a final grade in the subject mathematics. The grades are meant to be comparable across years.
- In a cross-sectional study of the environmental awareness for 15-year old students.
- When using a cognitive scale to screen for dementia among elderly people.

Maximum marks: 2

10 Rater reliability 1

Two raters rate the essays of 50 students with pass (score 1) or fail (score 0). Which of the following methods would be most appropriate to quantify the consistency of the ratings?

Select one alternative:

- Calculate the proportion of agreement while adjusting for chance agreement
- Calculate the Pearson correlation between the test scores from each rater
- Calculate the mean score for each rater
- Calculate the coefficient alpha for the sum of the rater scores

Maximum marks: 2

11 Statistics 1

X and Y are two random variables with variances $\text{Var}(X) = 10$ and $\text{Var}(Y) = 5$.

a) Assume that X and Y are independent. Present the expression of the variance of $Z = X - Y$ and calculate it. (1p)

Enter text here

b) Assume that $\text{Cov}(X, Y) = 1$. Present the expression of the variance of $U = X + Y$ and calculate it. (1p)

Enter text here

Maximum marks: 2

12 Statistics 2

The variance of a random variable X is $E[(X - E(X))^2]$. Show that $E[(X - E(X))^2] = E(X^2) - [E(X)]^2$.

Fill in your answer here

Maximum marks: 2

13 Coefficient alpha 2

Coefficient alpha is defined as:

$$\alpha = \frac{m}{m-1} \left[1 - \frac{\sum_{j=1}^m \sigma_{X_j}^2}{\sigma_Y^2} \right]$$

The following proportions correct were observed for three binary items: 0.7, 0.5, 0.6. The sum score variance was 1.5. Show that the observed coefficient alpha is equal to 0.8 and give an interpretation of the result.

Fill in your answer here

Maximum marks: 2

14 Covariance 1

The following covariance matrix was observed for two tests where one measured mathematics and one measured reading literacy:

$$\begin{pmatrix} 9 & 3 \\ 3 & 16 \end{pmatrix}$$

Based on this covariance matrix and possible calculations from it, do you consider mathematics literacy and reading literacy to be highly related? Justify your answer and list the assumptions made.

Note that the correlation between two random variables X and Y is $\rho_{X,Y} = \frac{\text{Cov}(X,Y)}{\sigma_X \sigma_Y}$.

Fill in your answer here

Maximum marks: 2

15 Standard error of measurement 1

The standard error of measurement is equal to $\text{SEM}(X) = \sqrt{\sigma_X^2(1 - \rho_{X,X'})}$. The reliability of

the sum score of a test was estimated to be 0.75 with sample variance of the sum scores equal to 16. An individual received a sum score of 11. Calculate an approximate 95% confidence interval for the true score of this individual. Give an interpretation of the interval and state the assumptions that are implicit in the analysis.

Fill in your answer here

Maximum marks: 2

16 Random variable 1

Consider the discrete random variable X that can take the values 0, 1 and 2 with probabilities:

$$P(X = 0) = 0.6$$

$$P(X = 1) = 0.3$$

$$P(X = 2) = 0.1$$

a) The expected value of X is .

b) The median of X is .

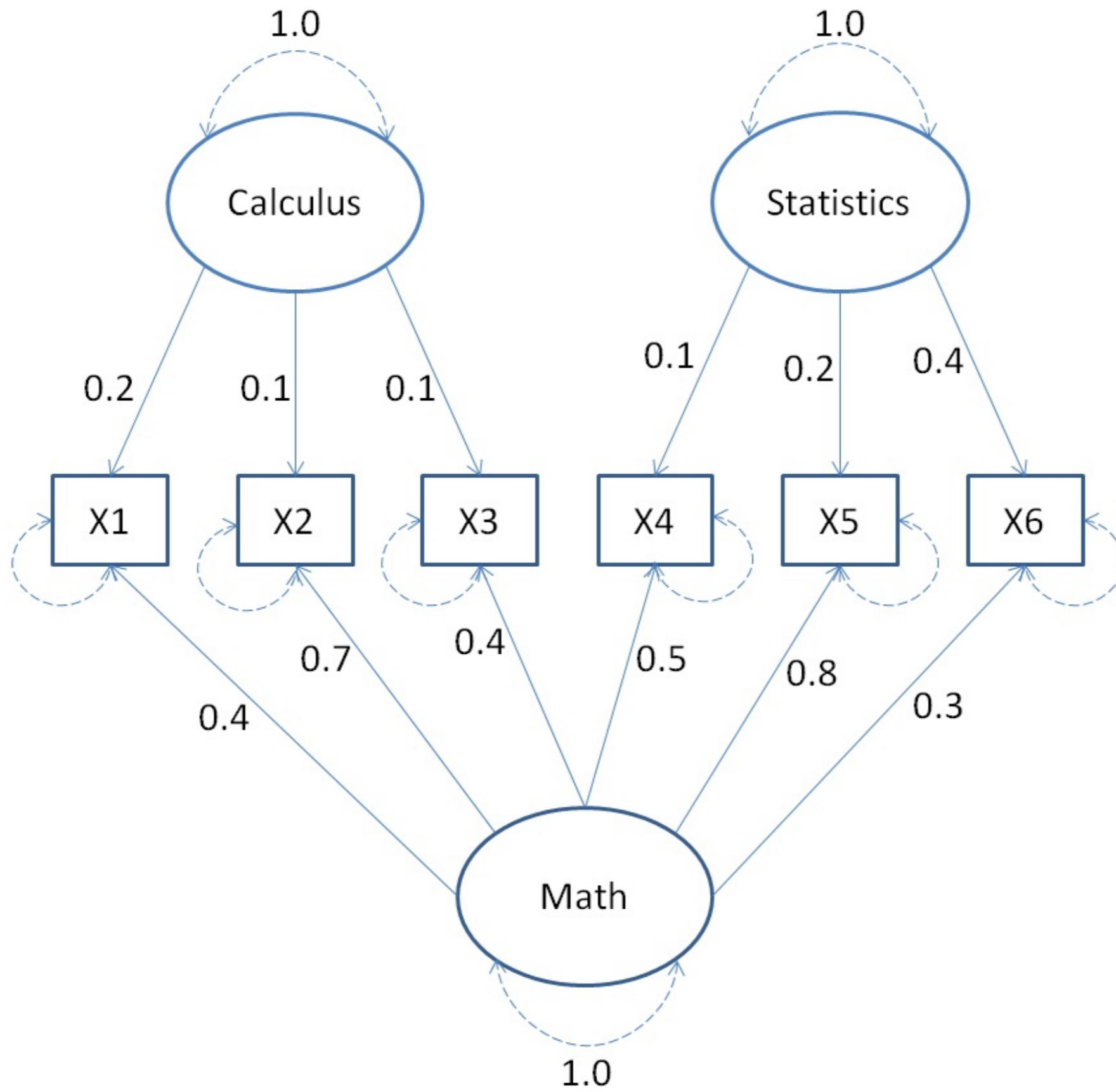
c) The mode of X is .

d) The variance of X is .

Maximum marks: 2

17 Factor model 3

Consider the graph below and answer the following questions.



a) What are the factors (i.e. latent variables) in the model? (1p)

Fill in your answer here

b) What is the correlation between the variables Math and Calculus according to the model? (1p)

Enter text here

c) What is the equation which describes the model for the item score X1? Write down the equation with an explanation of the parameters and variables included. (1p)

Enter text here

d) What is the correlation between item scores X1 and X4 according to the model? (1p)

Enter text here

18 Validity 3

A university entrance exam was used as a selection device for university. The exam was designed to measure aptitude for university studies. A study was conducted to investigate the predictive validity of the exam scores with respect to a measure of successful completion of courses at university (RELPREST). As a control variable, the GPA from high school (HSGPA) was also used. In the table below, the correlations between the exam scores/HSGPA and the variable RELPREST are given.

Variable	Correlation
HSGPA	0.56
Exam score	0.38

What is your assessment of the validity of the university entrance exam based on this study, in relation to the use of the exam scores for university admissions? In your response, you should consider how the criterion used is defined and what level of evidence the study can be considered to provide for its intended use. (Maximum 200 words.)

Fill in your answer here

Maximum marks: 4

19 Factor model 2

The following output was obtained from estimating a single factor model of five Likert-scale items from an attitude scale.

	Factor loading (λ_j)	Uniqueness (Ψ_j^2)
Item 1	1.00	4.00
Item 2	1.50	4.00
Item 3	1.50	3.00
Item 4	2.00	3.00
Item 5	3.00	5.00

The model gave GFI = 0.92, RMSEA = 0.07 and a mean of residuals of 0.00.

a) Does the model fit the data well? Justify your answer. (1p)

Fill in your answer here

b) The coefficient omega is defined as:

$$\omega = \frac{\left(\sum_{j=1}^m \lambda_j\right)^2}{\left(\sum_{j=1}^m \lambda_j\right)^2 + \sum_{j=1}^m \Psi_j^2}$$

Calculate coefficient omega from the output provided. Comment on the reliability of the sum score in light of the result and provide a motivation for your answer. (2p)

Enter text here

c) Assume that a single factor model is appropriate. Which item is contributing the most to the reliability of the sum score and which item is contributing the least? Justify your answers. (2p)

Enter text here

Maximum marks: 5

20 **Validity 2**

A scale is developed to measure panic disorder with the intended purpose to utilize the scale to identify individuals who have panic disorder. The scale consists of Likert items. According to the underlying theory of panic disorder, it is a unidimensional attribute. The theory also states that panic disorder is distinct from depression and is expected to have differences in severity based on gender.

a) List two different implications for the sum scores of the scale, based on the description of the attribute panic disorder given above. (2p)

Fill in your answer here

b) The scale was administered to a random sample from the population of Norwegian high school students. The students also responded to a short screening scale for depression which has been validated as a general measure of depression in the population considered. As part of the study, student demographic variables are also collected. Describe the validity evidence that can be gathered from this study and describe the statistical analyses required for inferring this evidence. Also describe the limitations to the validity evidence that can be gathered. (Maximum 200 words.) (3p)

Enter text here

Maximum marks: 5